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Deliverable D 2.2

TrainDy, Sensitivity Analysis

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Report contributors		
Name	*Beneficiary Short Name*	*Details of contribution*
Stefano La Rovere	NIER	Definition of the methodology for the sensitivity analysis. Discussion of results. Production of the deliverable.
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1 Introduction

This deliverable concerns the sensitivity analysis performed on the TrainDy software within the Marathon2Operation (M20) project.

1.1 M20 project

To achieve the objectives of the European Commission white paper on Transport 2011 aiming at a 30% shift to rail of freight transportation over 300km by 2030, the rail freight transport market share has to increase strongly. The market **requirement** are competitiveness, reliability, flexibility, frequency and information. The previous FP7 MARATHON project [7] demonstrators have shown the feasibility of 1500m long coupled heavy trains with distributed power of two Traction Units (TU) running safely on the French network. Building on that, M2O intends to extend the possibilities to four locos as distributed power system in collaboration with S2R-IP5-CFM-01-2018 project [8]. To reach that goal, M2O sets up a reliable transfer of data and commands between the locos based on GSM-R technology. The proposed solution is aimed to be compatible with various suppliers of GSM-R and its safety has to be analyzed and assessed.

The solution is integrated in the train Distributed Power System (DPS) and the safety of the system is studied to cope with the various operational situations. Having set the radio communication system, the project aims to analyze its usage varying the main possible consists characteristics in terms of speed, type of wagons, acceptable load and its distribution along the train by using TrainDy [6] simulations to ensure that it may run safely. The safety analysis will ensure that the various hazards have been correctly taken into considerations while performing the simulations and that the adequate mitigations have been elaborated Jointly with the CFM Partners of S2R-IP5-CFM-01-2018 project. Finally, two demonstrators have been foreseen whose safety will be studied and assessed to get the authorities green light to perform the tests on the rail network.

1.2 Why a sensitivity analysis is needed

In general, more than two TU at the head of the train could exceed the maximum effort at the coupler. For trains longer and heavier, multiple distributed traction is necessary, i.e. one TU at the head and another or others in the middle or at the end of the train pushing the load instead of pulling it. Traction and braking on all the locos shall be coordinated by a safe and reliable communication between TU that needs to operate with just one driver. The introduction of automated "distributed power" technology enables the operation of longer trains. In many cases, distributing tractive effort along the train will result in lower in-train forces. However, a number of parameters (only partially under control) could affect the longitudinal forces experienced during the different operational conditions, by different trains. For this reason, it is essential that train forces are correctly analyzed and managed in distributed power operations.

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The safety analysis to be performed during the M2O project for the demonstrator (Deliverable 3.2) will be partially based on the results coming from simulations performed by TrainDy.

TrainDy is the UIC certified software currently used to address the Longitudinal Train Dynamic (LTD), *i.e*. the relative motion of adjacent vehicles running in track direction. It is capable to numerically model the time evolution of air pressure in brake pipe and brake cylinders (i.e. pneumatic problem) and the relative motion between consecutive wagons (i.e. mechanic problem). It is employed to compute in-train forces for different operational scenarios and considering different types of wagons: two axle wagons, bogie wagons, high resistance (or high performance) wagons (i.e. characterized by high admissible in-train compressive forces or longitudinal compressive forces). Since in-train forces are affected also by other parameters, like train mass/length, type of train operation, speed, mass distribution, type of brake block (cast iron, type LL, type K) or disc brake technology, braking equipment (empty/load or auto-continuous), track gradient and others, it is important to ensure that the uncertainties on them do not result in significant differences in simulations output. For this reason, a sensitivity analysis has been proposed to augment the trustworthiness of TrainDy simulations and their extrapolation potential.

For instance, the results coming from a proper sensitivity analysis allow:

- to verify key-assumptions of the simulations to be performed for demonstrator trains, e.g. concerning the number of trains with different composition (in terms of number, type and mass of their wagons) to be considered as a sample representative of the "real distribution";
- to identify the key parameters driving LTD, by assuming a (fictitious and realistic) uncertainty, to be considered in the simulations for demonstrator trains;
- to identify interactions among parameters (i.e. non-linear change of the outcome for the concurrent changes of input parameters, in different numbers and combinations), if relevant;
- to compare long trains with shorter ones, in terms of sensitivity of longitudinal forces;
- to justify disagreement between experimental and simulations results.

1.3 Objective of the sensitivity analysis

This deliverable describes the general procedures and tools developed for performing a sensitivity analysis on LTD simulations made by TrainDy and provides exemplificative results accounting for the uncertainty on a subset of parameters.

The final objective is to support the TrainDy simulations that will validate the Longitudinal Train Dynamic of the demonstrator trains (out of scope of this document and available in D3.3). For this purpose, it addresses different methodological approaches to measure the sensitivity outcomes and provides the results obtained for (fictitious and realistic) uncertainties assigned to a subset of parameters.

1.4 Technical Vs Operational Train Parameters

When considering the parameters potentially affecting the Longitudinal Train Dynamic, a major distinction should be made between[1] **Technical Parameters** and **Operational Parameters**, following the terminology of current UIC Leaflet 421 [1].

Technical Parameters are usually intrinsic train characteristic (e.g. the brake pipe diameter, or the initial pressure in brake pipe). On average, they present "physical" small uncertainties (e.g. due to tolerances in the manufacturing process or measure errors or aging of equipment) on which Railway Undertakings have not (or very low) control.

Operational Parameters are factors on which Railway Undertakings have full or partial control (e.g. concerning track characteristic, train operations, train system setting). They can usually experience significant variations and contribute to set completely different braking simulation scenarios.

According to §1.3, the present Sensitivity analysis supports the LTD simulations performed for the demonstrator trains. Specifically, it is focused on the methodological approaches to measure the sensitivity of LTD simulation outcomes, applied to the Technical parameters.

The LTD simulations for the demonstrator trains will refer to the Operational parameters, whose range of variability defines the limits of their specific application. Within this context, a complete set of Operational parameters should be considered.

1.4.1 Technical Parameters

The following tables list all the technical parameters considered throughout the analyses, preliminarily identified as the most potentially influential in LTD [9]. They concern: Track slope and speed (Table 1), Braking (Table 2), Control Valve (Table 3), Brake Pipe (Table 4), Communication (Table 5).

Each table report the name (parentheses enclose the TrainDy keyword) of the related parameters and their associated Gaussian uncertainties (in terms of mean value ($\bar{x}$), standard deviation (s) and type (p: proportional; a: absolute); the last column provides references to sources and/or remarks

[1] It has to be noted though, that the line dividing technical from operating parameters is actually pretty blurred and ultimately depends on the analysis context. For instance, the "emergency braking starting speed" can be considered both a technical parameter when the object of the analysis is to investigate the importance of the +/- 3% uncertainty due to the tachymeter specifics, or an operating one, when the analysis focus is to investigate what happens when the speed shifts from 30km/h to 60km/h for example.

on the specified values. When a parameter is indicated as proportional ("p") it means that the standard deviation is expressed as a percentual, and the mean value is set to 1. This is done since the mean value can vary for different wagons. Conversely, when a parameter is indicated as absolute ("a"), both the mean value and the standard deviation are reported in the table as absolute value (equal for all wagons).

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Table 1 - Track and speed parameters

Parameter	$\overline{x}$	s	Type	Reference / remark
Slope	0	1/3	a	Usually, Infrastructure Manager provides to Railway Undertakings the track slope with a maximum error of 1‰
Speed (stspd)	1	0.01	p	Usually, the maximum tachymeter scatter is 3%.

Table 2 - Braking parameters

Parameter	$\overline{x}$	s	Type	Reference / remark
Max pressure at brake cylinder (bcExpTp)	1	$8.77 \cdot 10^{-3}$	p	According to [1] (p.1.7: 3.8 ± 0.1 bar)
Cross section brake cylinder (bbCylSection)	1.0003	$2.05 \cdot 10^{-4}$	p	Typical tolerances of a pneumatic cylinder
Rigging ratio (bbRigRatio, dbSRigRatio)	1	$\sqrt{2} \cdot \frac{0.01}{3}$	p	A maximum variation of 1% of each leverage length has been assumed. $\sqrt{2}$ is the result of manipulation of random variables.
Force applied by the brake rigging return spring (bbFF)	1	0.1/3	p	It is the parameter F_F in [3] (e.g. p. 33). Variability has been estimated according to maintenance experience.
Counteracting Force of the brake rigging regulator (bbFR)	1	0.1/3	p	It is the parameter F_R in [3] (p. 9). Variability has been estimated according to maintenance experience.
Mean efficiency of the rigging (bbRigEff, dbSEffic)	1	0.08/3	p	See [5]

Table 3 - Control Valve Parameters

Parameter	$\overline{x}$	s	Type	Reference / remark
Time to reach 95% of maximum pressure in BC, for braking position G (ft95)	24	2	a	For 95% this time is between 18 and 30s, according to [1]
Time to reach maximum air pressure in braking position G (ft100)	0.2	0.02/3	ap	According to the experience gained during *TrainDy* validation, value for 100% is equal to 95% time multiplied by 1.2. It has been added an uncertainty of 10%.
Pressure drop in brake pipe to activate accelerating chambers (dpBPPABrCyl).	1	0.1/3	p	This parameter is managed by a gradient according to [1]; according to TrainDy model, a pressure variation is used. A variation around reference value of ±10% is assumed.
Pressure drop in brake pipe to activate brake cylinder (dpBPPAAcCha)	1	0.1/3	p	This parameter is linked to control valve transfer function [1]. For this parameter, a dispersion of ±10% around mean value has been also assumed.

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Parameter	$\overline{x}$	s	Type	Reference / remark
Pressure in brake cylinder for application stroke (asPresBrCyl)	1	0.1/3	p	These parameters are used in *TrainDy* model to manage the first part of brake cylinders filling ("first time braking"), independently from braking regime. For these parameters, a dispersion of ±10% around mean value found during *TrainDy* validation has been also assumed.
Time for "application stroke" phase (asTime)	1	0.1/3	p	
Pressure in brake pipe for "application stroke" phase (genPipe)	1	0.1/3	p	
Pressure in brake cylinder for "in-shot function" phase (ifPresBrCyl)	1	0.1/3	p	
Time for "in-shot function" phase (ifTime)	1	0.1/3	p	

Table 4 - Brake Pipe Parameters

Parameter	$\overline{x}$	s	Type	Reference / remark
Brake pipe diameter (brakePipeDiam)	1	$1.5 \cdot 10^{-3}$	p	Considering usual manufacturing tolerances of drawn pipes
Initial pressure in brake pipe (initpres)	1	0.01/3	p	In [4], there is a reference to an average value of 5 bar and to a scatter of ±0.05 bar.

Table 5 – Communication Parameters

Parameter	$\overline{x}$	s	Type	Reference / remark
Delay scatter in communication (ctrl)	1.7	0.17	a	According to the expertise of Funkwerk. FFL4E experimental tests of May 2019, analyzed in October 2019 showed an average value of 1.6 and a standard deviation of 0.16. This data refinement was not available at sensitivity analysis production time.

1.4.2 Operational Parameters

The Operational parameters to be considered in the TrainDy simulations for the demonstrator trains could concern the Infrastructure, the Train operation and the Train systems settings.

Infrastructure Operational parameters will be defined with reference to the characteristics of the track selected for the test runs of the demonstrators (e.g. maximum and minimum slope).

Train operational parameters concern the different types of maneuvers (or train operations) that the train can experience, affecting the longitudinal forces. Two possible maneuvers are taken as reference in the present sensitivity analysis (see §1.5). A complete set of maneuvers will be defined for the TrainDy simulations to be performed for the demonstrator trains.

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The Train systems setting parameters could concern the Traction and Electrodynamic Braking (e.g. gradients of application and removal), the Distributed Power System, the Driver's Brake Valves, the Type of shoes, the Type of Couplings, the Control Valves (e.g. timings and pressure levels).

1.5 Reference Maneuvers

This sensitivity analysis takes as reference two possible emergency braking maneuvers:

- a standard emergency braking maneuver, i.e. commanded from an initial speed of 30km/h and going to zero;
- an alternative emergency braking maneuver, i.e. an acceleration from zero to 30km/h followed up by an emergency braking command. This maneuver will be referenced using the codification "N202".

It is worthwhile to mention that a train can operate in many more ways (e.g. first-time braking, service braking, full-service braking, etc.), nevertheless, the two train operations listed above are considered the most dangerous according to the experience of Railway Undertakings.

1.6 Deliverables Structure

The deliverable structure follows the step by step approach used for the analysis production:

- §2 describes the tools and methodology that were derived for the analysis;
- §3 provides the results coming from a preliminary exploration of TrainDy model behavior and technical parameter influence;
- §4 provides the results coming from the sensitivity analysis based on "realistic" uncertainties associated to the technical parameters;
- §5 provides the results of the investigation on the effect of the size of a trains' family;
- §6 concludes the deliverable and report the final considerations of the work.

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2 General concepts and Tools

2.1 TrainDy input/output manipulation

The basic goal of a sensitivity analysis is to measure the change in the model's outcome(s) due to changes of the input parameters because of (aleatory and epistemic[2]) uncertainty. The typical output is the ranking of input parameters according to their influence on the model's outcome(s). For the development of this sensitivity analysis, TrainDy software is considered as a "black box": only inputs and outputs were manipulated, whereas it was out of scope to investigate on possible parameters hardcoded into the software that may have significant effect on simulations outcome, since the software results have been validated by comparison against several experimental cases [6].

A python API (Application Programming Interface) was developed in order to deal with the set of ".txt" files and ".mat" files that constitutes respectively TrainDy inputs and outputs and to efficiently post-process the large quantity of data generated by the simulations. The more important API capabilities are the following ones:

- Automatic generation of TrainDy inputs, starting from a reference one, changing one or more parameters at the time;
- Automatic run of TrainDy software;
- Automatic reading of results and extraction of significant data;
- Medium-high level tools for post-processing (i.e. data manipulation, tables and plots generation).

Figure 2.1 displays a schematic description of the python API capabilities.

[2] No distinction is made between Aleatory (or statistical or stochastic) uncertainty, which is related to an inherent variation of a parameter (e.g. variability in geometric parameters due to manufacturing), and Epistemic (or systematic) uncertainty, which arises from imperfect knowledge or ignorance (e.g. poor understanding of physics phenomena).

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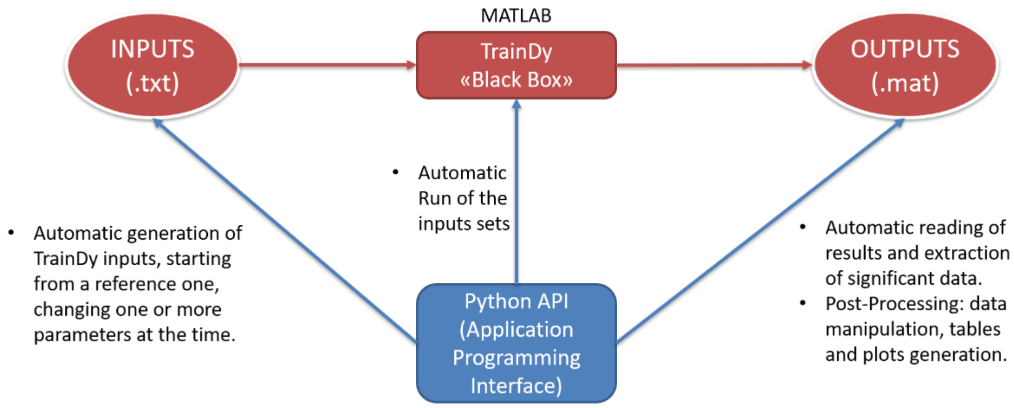

Figure 2.1 – Schematic description of python API capabilities

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2.2 Reference trains families

The "reference trains family" is a key concept to understand the following analyses results and it is also used in UIC Leaflet 421

In the real world, freight trains lack strict standardization: their total length and mass must remain inside specific ranges, but the number and type of wagons or the freight load distribution can wildly vary. This means that in order to investigate the longitudinal forces caused by a braking maneuver, one cannot simply identify a reference train, instead a "reference train family" has to be statistically generated.

The task of generating random trains is crucial for a statistical investigation on in-train forces of a freight train; this topic is, in turn, crucial for a risk assessment of new freight trains, as far as LTD is concerned. The adopted algorithm is compliant with the UIC Leaflet 421[3] ([1], Appendix B) procedure for the generation of random freight trainsets. The train mass, the traction unit, the wagon type, their number and their payloads are defined in terms of probability distributions, based on trains running on the railway network. Input statistic data for following results are extracted from the database of real running trains in Germany, provided to the University of Rome "Tor Vergata", by DB Systemtechnik, for the accomplishment of this Project. The procedure is described in the reports for the update of UIC Leaflet 421 and for the UIC Long Train.

Following what was done in the aforementioned UIC projects, each "reference family" generated for the sensitivity analysis was composed by 1000 trains randomly sampled from the database of freight trains currently running on German network.

As stated above, §5 provides the results coming from an additional study focused on the number of trains that are to be considered in LTD simulations in order to obtain statistically significant results (for given basic family and maneuver). It is possible to anticipate here that 1000 trains were found sufficient to obtain engineering acceptable results.

2.2.1 Basic families description

Three basic trains families are used throughout the sensitivity analysis:

- **S2RGH -** Trains in G with almost all wagons loaded at the same level; hauled mass is between 2500t and 5500t;

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[3] UIC Leaflet 421 [1] provides the requirements for the composition and braking of cross border freight trains according to their braking regime and maximum speed, in order to speed up operations at borders and transfer points.

- **400LL 300GP -** Once coupled the train runs in G; it is originally formed by a train in LL (average length 400m and hauled mass between 1200t and 1600t) and another in GP (average length 300m and hauled mass between 800t and 1200t);
- **4GP –** As above, once coupled, the train runs in G and it is formed by coupling together four trains originally running in GP having each sub-train an hauled mass between 800t and 1200t. The overall train length is 1500m.

2.3 Simulation results extraction: CDFs

The sensitivity analysis is focused on two main TrainDy outcomes:

- "Flong2", which contains all the minimum (in module) longitudinal force registered for each draw gear (or screw coupler) of a single train at every time step in the preceding 2 meters; the maximum positive force registered among all consecutive vehicles at all time steps is considered the worst traction (or tensile) force experienced by the single train.
- "Flong10" , which contains all the minimum (in module) longitudinal force registered for each couple of buffers of a single train at every time step in the preceding 10 meters; the maximum (in module) negative force registered among all consecutive vehicles at all time steps is considered the worst compression (or compressive) force experienced by the single train.

Once the simulation is completed, both for compression and traction forces, it is possible to generate an experimental CDF (Cumulative Distribution Function) starting from the frequency histogram built from the 1000 trains worst forces. When creating the initial histogram, a sufficiently high number of bins have to be chosen since each of them will translate into a single point of the CDF: 800 bins were used during this analysis since it was shown that adding more was not resulting in notable changes of the final CDF shape.

Figure 2.2 shows an outcome example of this results extraction process.

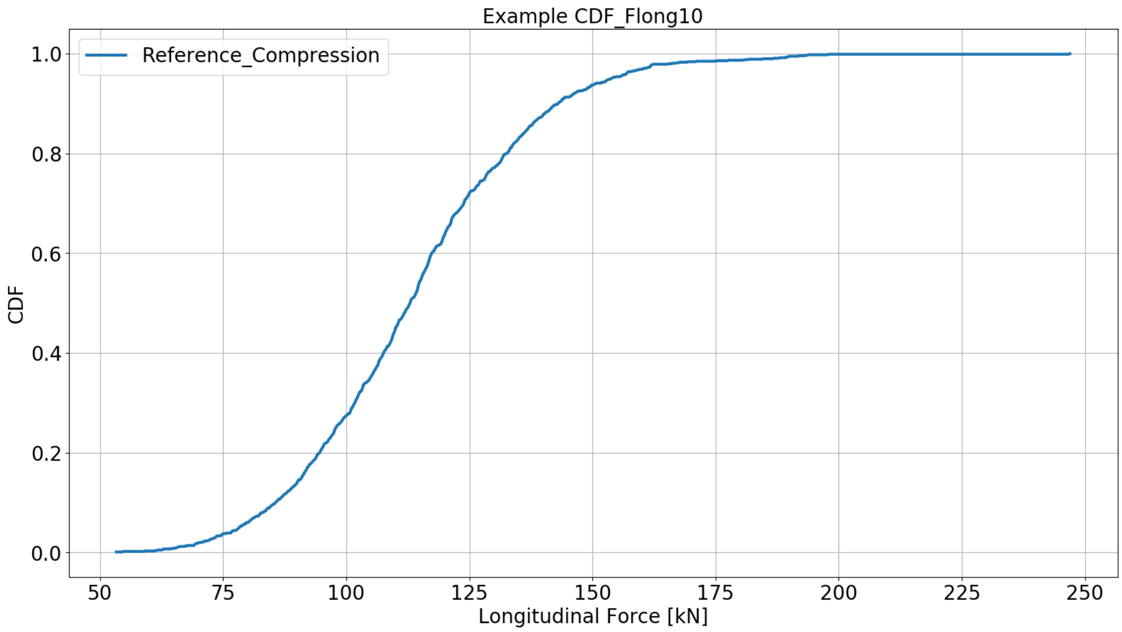

Figure 2.2 – Example of a CDF generated from TrainDy simulation outcome.

2.4 UIC Admissible longitudinal force

CDFs in the following sections have been generally built on the longitudinal forces (i.e. traction and compression) experienced by the single trains of a family. From these results though, the UIC force ratio can be also derived, that is, the highest ratio between the longitudinal force experienced among all consecutive vehicles of a train and the admissible force that the wagon can withstand.

The admissible or permissible in-train forces differ for compressive in-train forces and tensile in-train forces. Admissible longitudinal compressive forces (LCF) are relevant for the wagon derailment risk, whereas admissible longitudinal tensile forces (LTF) are relevant for the train disruption risk. UIC Leaflet 421 gives a procedure to determine a suitable value of admissible LCF, applicable for assessment procedures based on statistical simulations. The method is based on the admissible LCF computed following UIC Leaflet 530-2. This value of force is then incremented considering: i) a radius of curvature bigger than 150m; ii) the wagon payload; iii) the buffer plate radius.

2.5 Sensitivity measures

Every sensitivity analysis requires one or more scalar output(s) in order to measure the influence of the uncertainty assigned to input parameters on the uncertainty resulting from the simulations

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outcomes and to build a sensitivity ranking of parameters. The selection of the measuring parameter is a crucial step of the analysis since different indicators provide different information about the simulation outcomes, leading to different rankings of the input parameters.

As previously introduced, the output of LTD simulations for a family of trains made by TrainDy is not a single value, but a cumulative distribution instead. This means that proper indicators have to be developed in order to compare results coming from different LTD simulations.

The first approach that was followed was to try to fit the experimental CDFs with analytical distributions, like the Gaussian or Rayleigh one, with the idea of considering their characteristic statistical parameters as indicators to use in the comparisons (e.g. the mean and variance for a Normal distribution). An example of a similar attempt is shown in Figure 2.3: on the left the experimental frequency histogram and CDF fitting curves are reported, while on the right one can see the Quantile-Quantile plots both for the Gaussian and Rayleigh fitting. This kind of graphs help to understand how good a fit is: the more points lay on the main diagonal, the better is the fit.

Based on the obtained results, this approach was discarded since it was often difficult to clarify which was the best statistical distribution to use for the fitting and, in general, the tails of the different distributions failed to properly fit the real data. For these reasons, specific "non-parametric" sensitivity measures are used, i.e. independent from assumptions on statistical distributions.

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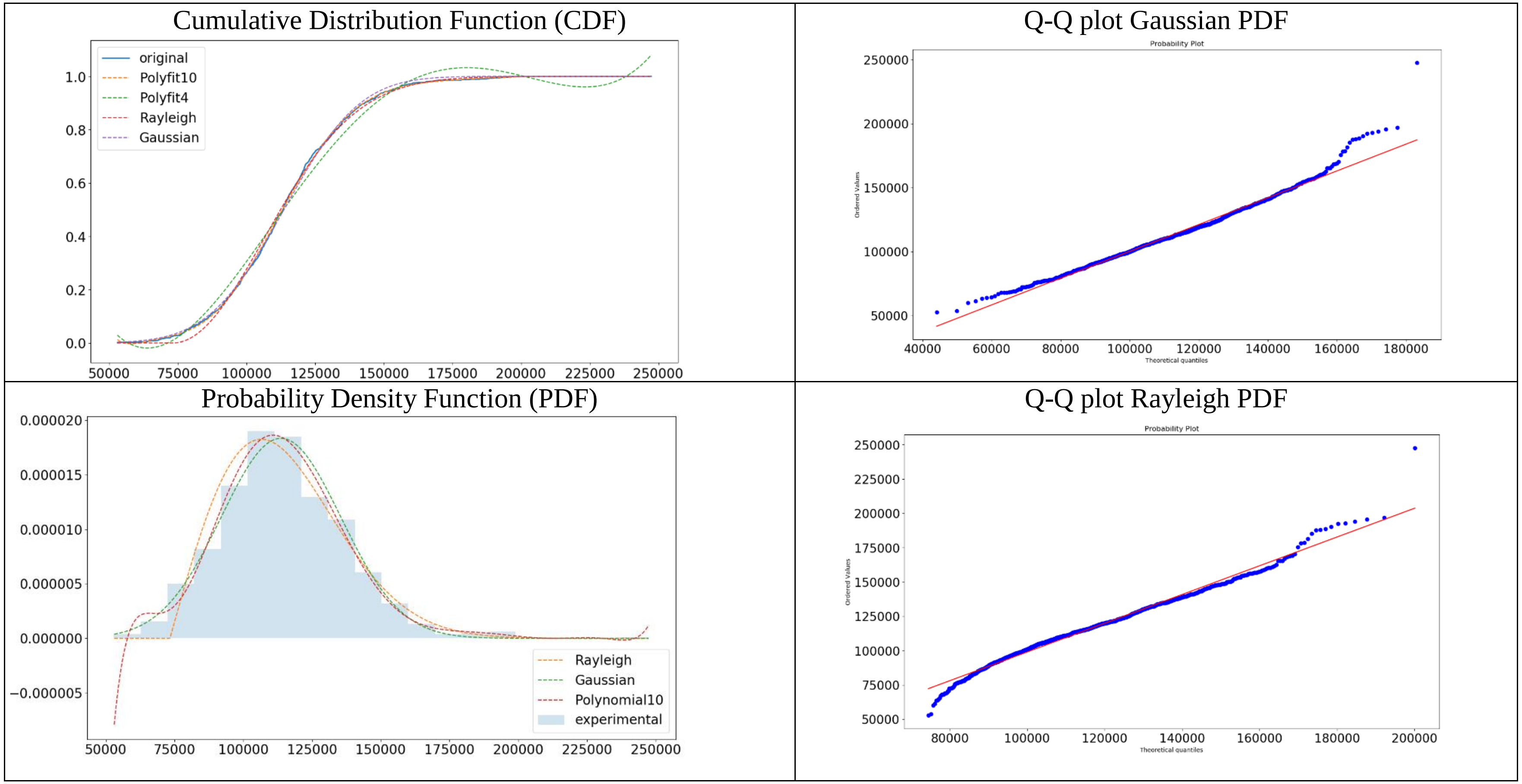

Figure 2.3 – Fitting attempt on an Experimental CDF both with Rayleigh and Gaussian distributions

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2.5.1 Differential: Absolute and Algebraic Areas

The simplest way to quantify the difference between two CDFs is to measure their differential area as highlighted in Figure 2.4.

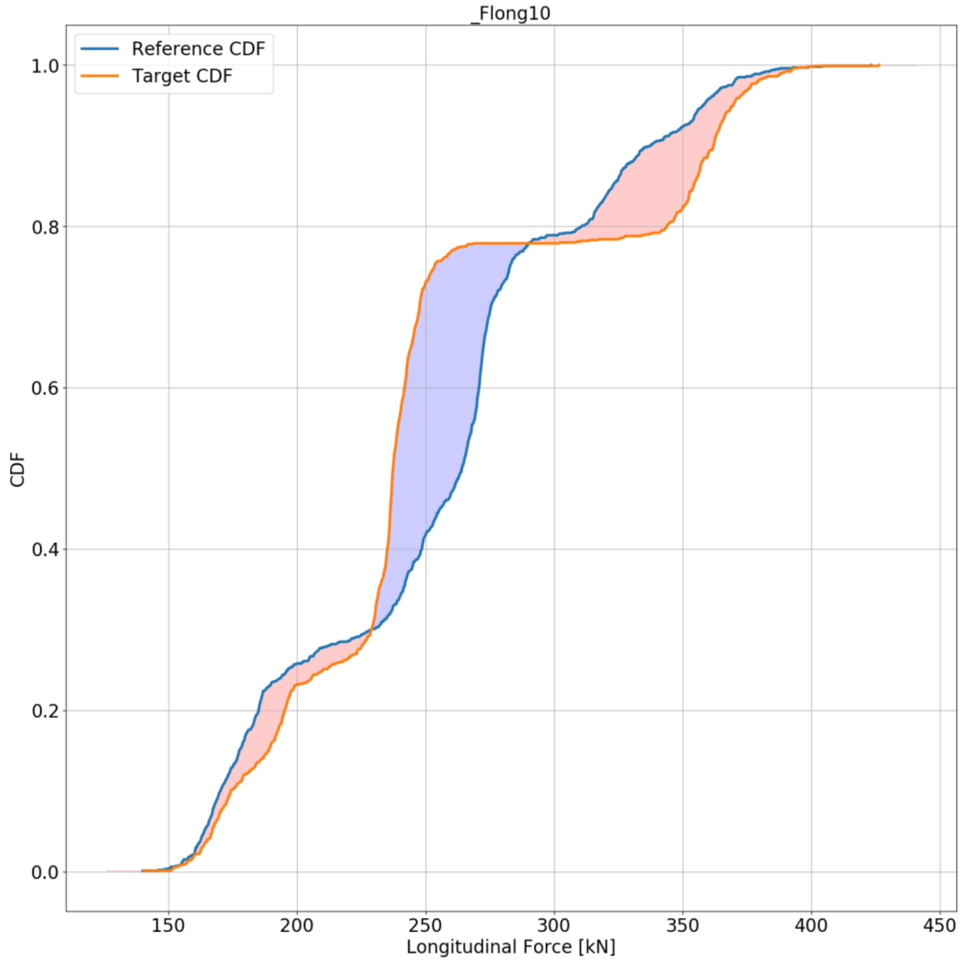

Figure 2.4 – Differential Areas. In red the "negative" area while in blue the "positive" one

In the picture, a distinction is made between positive (in blue) and negative (in red) areas. This is due to the fact that when considering one of the two CDFs as the reference and the other as the target (obtained from the variation of one or more parameters from the reference), it is possible to establish a convention on the areas sign. That is, when the target CDF "lies under" the reference one (e.g. for a specific outcome value), a single train belonging to the reference family will have a higher probability to present a smaller or equal force to that specific one, hence the negative performance of the target family. For the positive area the same reasoning applies, only in a specular way.

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Once a convention is finalized, calling P the positive area and N the negative one, it is possible to define two different indicators[4]:

- "Absolute Area" = |P|+|N|, which is a measure of how much the target and reference curves globally differ.
- "Algebraic Area" = P+N, which indicates if the target performed better or worse than the reference in terms of longitudinal forces and to quantify this improvement or worsening.

2.5.2 Lower Force Probability Differential

A possible indicator to be used in the comparison of a reference and a target trains family, called the Lower Force Probability Differential (LFPD), is introduced in the following paragraphs.

If two trains are extracted respectively from the target and reference family, it is possible to determine the probability that the target train will present lower longitudinal force than the reference one. This probability has been called the Lower Force Probability (LFP). A mathematical description of how to compute such probability is presented in Annex A.

Comparing two trains sampled from the same distribution , the probability of one train having lower forces than the other one is always (independently from the distribution) 50%, as demonstrated in Annex B.4. Hence, the LFPD is defined as the difference between the LFP for the target-reference couple and 0.5, i.e.:

$$\mathrm{LFPD} = \mathrm{LFP} - 0.5$$

LFPD carries a similar information to the Algebraic Area one, since both focus on better or worse global performance of the target train (in terms of longitudinal force) respect to the reference. The LFPD though it is not a mere number, but express a probability that can be easily conceptually understood and may have other applications than solely the creation of an importance ranking.

With reference to the objective of the sensitivity analysis, LFPD can be used to compare how the variation of one parameter affects two different trains families (that would not be possible using the algebraic area without some kind of normalization).

Some examples of LFPD computation are showed in Figure 2.5, where, as mere example, the cross section of the brake cylinders is changed with respect to the reference value: on the right it assumes a lower value and on the right it assumes a higher value

[4] Only areas that were computed starting from the same reference CDF can be properly compared.

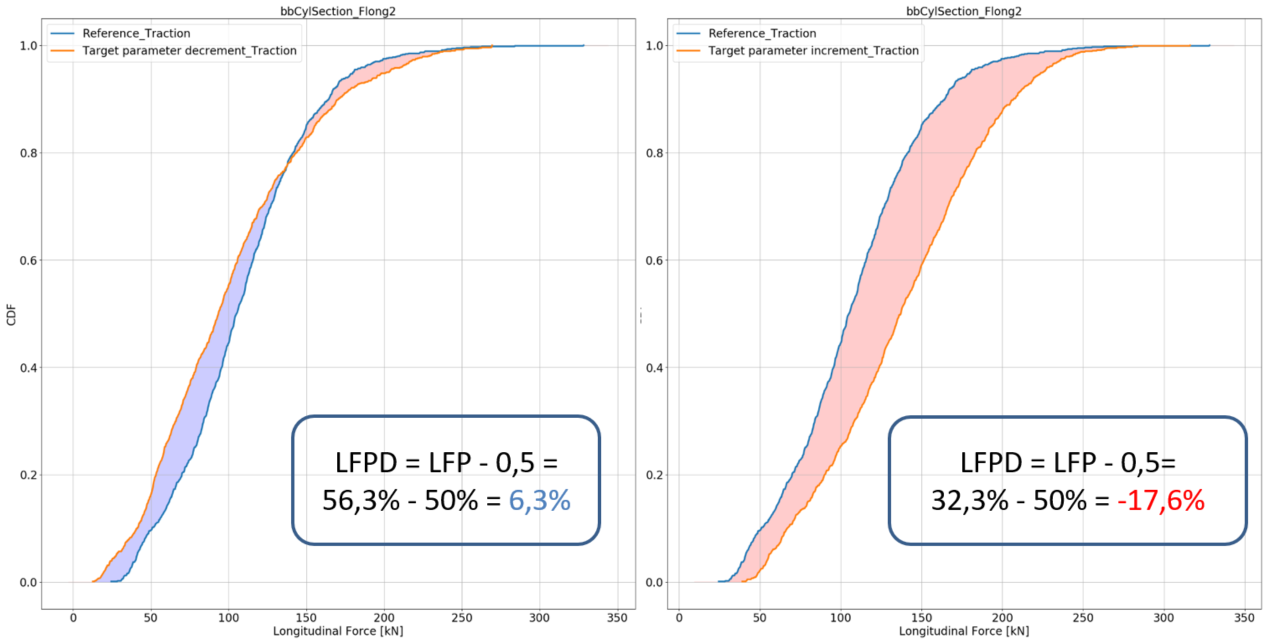

Figure 2.5 – Examples of LFPD computation

It is important to notice that LFPD averages the behavior of the (target) family (with respect to the reference one) on the whole force spectrum. As an example, considering the left graph, the LFPD is positive, indicating that, on average, the target trains family performs *better* than the reference one in terms of Longitudinal Forces, i.e. it has statistically *lower* Longitudinal Forces. However, being interested in high longitudinal forces (as for safety assessment), a train extracted from the target family actually have a higher probability to present larger forces with respect to one extracted from the reference family (it happens for F > 150 kN in the exemplificative case), i.e. the target trains are worse than the reference trains, if only high forces are considered.

For this reason, the LFPD definition is generalized in §B.3, in order to extend its applicability also to a specific forces range.

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3 Preliminary exploration of TrainDy model

3.1 Objective

This preliminary sensitivity analysis is a general exploration of TrainDy model response, varying all the technical parameters in §1.4 (namely Table 1, Table 2, Table 3 and Table 4).

This preliminary sensitivity analysis is performed in order to:

- get a general idea of the model behavior and a first glance at parameters variation influence on longitudinal forces;
- develop and test methods, techniques and indicators that were later applied to the other foreseen analyses;
- explore how a fixed percentage variation of a specific parameter can have a different effect on long trains respect to short ones.

3.2 Methodology

For this preliminary exploration, two reference trains families were considered: 400LL 300GP for long trains and S2RGH for short ones. See §2.2.1 for additional details on these families. Both these families are considered to perform both the standard and alternative emergency braking maneuvers introduced in §1.5.

The Lower Force Probability Differential (LFPD) is used as sensitivity measure.

With reference to the different techniques developed for sensitivity analysis, briefly introduced in the Appendix A (Importance and sensitivity analysis), the "Finite change" approach is adopted.

All parameters listed in §1.4.1 were varied of $\pm 30\%$ for both families in the two braking maneuver configurations. The assignment of the same fictitious uncertainty on all the parameters aims at investigating their influence on the outcomes because of the "structure" of the model. Significant changes of parameters are introduced in order to get significant changes from the reference family CDF shape and to test TrainDy model response on extreme conditions. Realistic assumptions for the uncertainty to be assigned to each parameter are made in section §4, therefore the results of this analysis are just to fulfill the objectives described in §3.1

3.3 Results

The total number of TrainDy runs for this preliminary analysis were:

$$4\ families \cdot 1000\ trains \cdot (\sim)20\ parameters \cdot 2\ (incr/decr)\ \approx 160k$$

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The huge amount of output data has been post-processed by the python API and to efficiently present the results, two type of plots were generated: CDFs plots and Tornado plots.

CDFs plots contain a high degree of information, but a large number of them is necessary to fully represent the results. Hence, in order to summarize the analysis, the Tornado plots were created.

3.3.1 CDFs Plots

Figure 3.1 shows an example of CDFs plot, for the variation of brake pipe diameter.

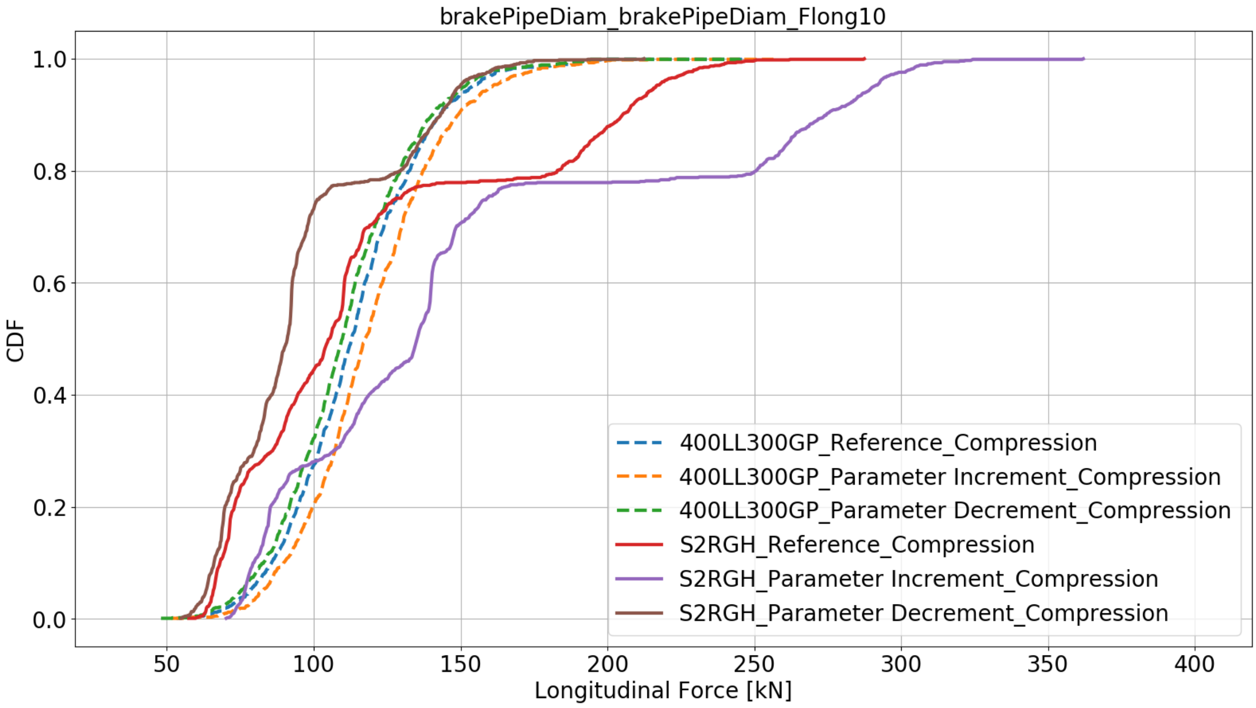

Figure 3.1 – Example of CDFs plot

These plots allow to have qualitative comparison between long (400LL300GP) and short (S2RGH) trains behavior and a qualitative idea of the different parameter variations influence on the trains. For instance, in Figure 3.1 three CDFs are plotted both for the short train (S2RGH) and for the long one (400LL300GP). Those are the CDFs associated to the compression forces experienced by the reference configuration of each family and the ones derived by the increasing and decreasing of the brake pipe diameter. In this particular case, it can be observed how this parameter appears to have a higher impact on the long train respect to the short one and how both trains families experience in general higher compression forces when the diameter is incremented.

A similar plot is produced for each maneuver (see §1.5), for each type of force (compression or traction) and for each (Technical) parameter. The complete list of plots can be found in the "M2O D2.2 Extended results.pdf" attachment that completes this deliverable as described in the Annex.

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3.3.2 Tornado Plots

As previously anticipated, tornado plots allow to summarize effectively the results of the entire analysis. In fact, each tornado plot contains information about all the parameters increments and decrements for a specific force type and maneuver, for a total of 4 plots. As illustrated in Figure 3.2. the histogram is produced in the following way:

1) All CDFs associated to reference and derived trains families are computed;
2) For each derived family (i.e. a family derived by the decreasing or increasing of single technical parameter) the LFPD is calculated respect to its reference one and reported in the graph as a histogram bar.

The short trains results are reported in blue , whereas the long trains ones are in red. For all families, a full color is used for results relative to parameters increment, while a shaded one has been chosen for the decrement.

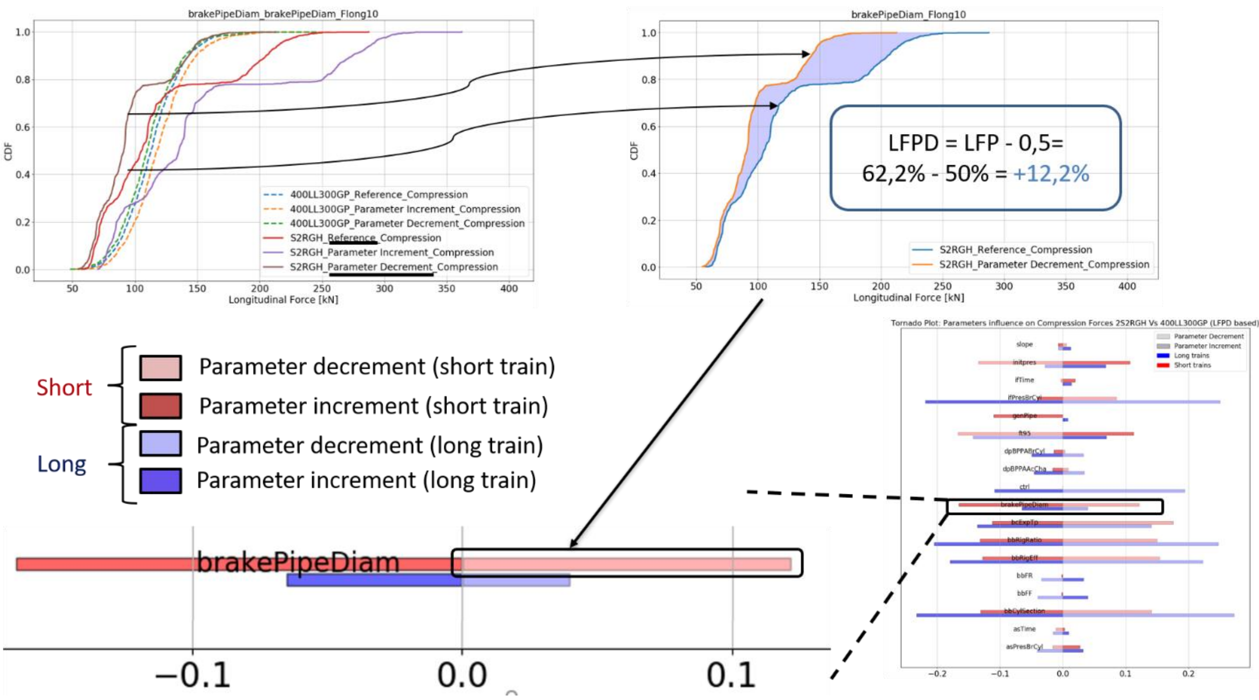

Figure 3.2 - Explanation of Tornado plot creation and example

This kind of plot immediately allows to capture which are those parameters that have a bigger impact on LTD simulation outcome (i.e. a large LFPD) and if their variation has a positive or negative effect in terms of reduction of longitudinal forces (i.e. looking at the LFPD sign). The results coming from the analysis are hereafter reported in Figure 3.3, which shows the tornado plot obtained for the S2RGH Vs 400LL 300GP trains families, and in Figure 3.4 , which shows the one obtained for the N202 S2RGH Vs N202 400LL 300GP trains families.

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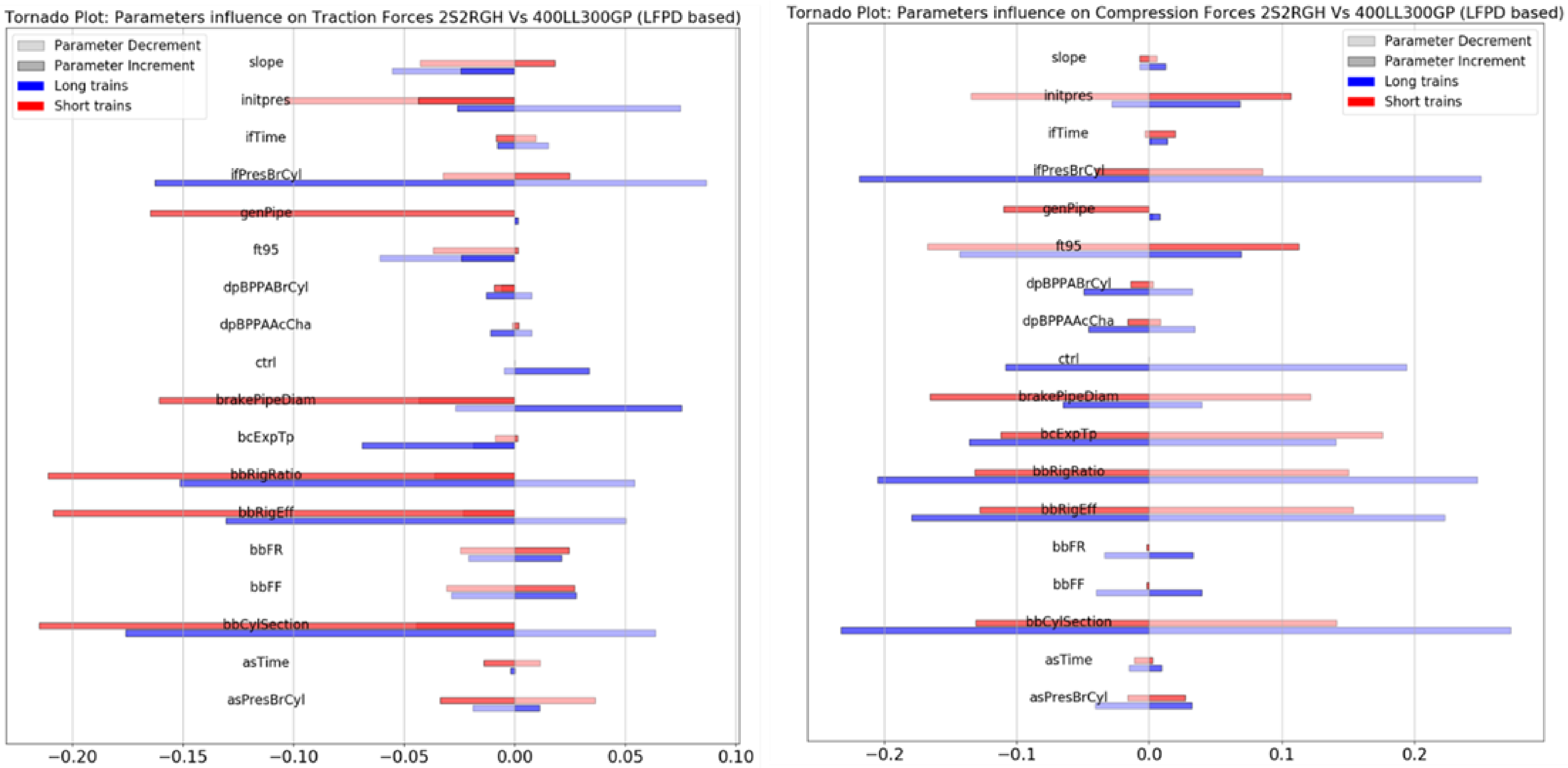

Figure 3.3 - Tornado plot comparing the results of S2RGH Vs 400LL 300GP families

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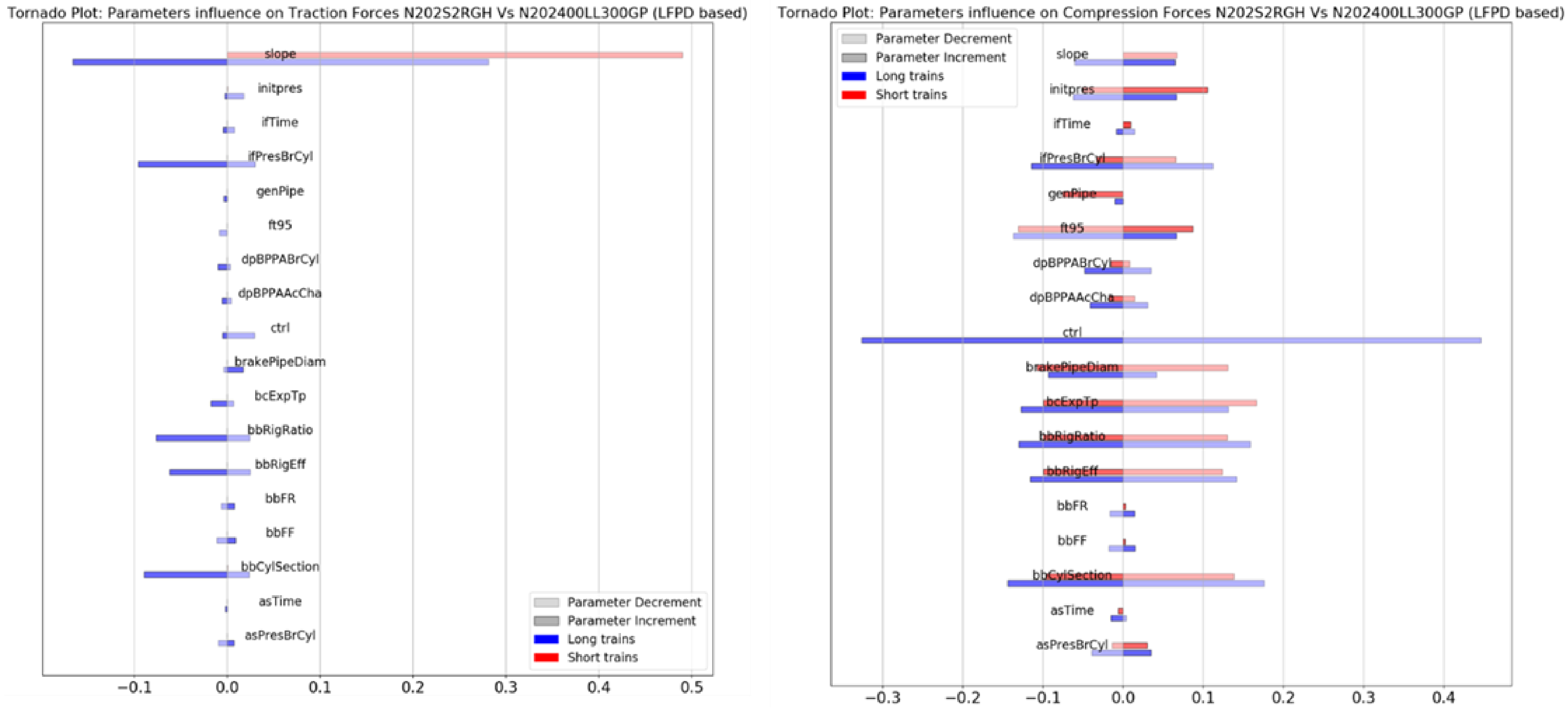

Figure 3.4 - Tornado plot comparing the results of N202 S2RGH Vs N202 400LL 300GP families

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3.3.3 Final Considerations

As previously explained, the scope of this preliminary analysis was to explore the model and trains behaviour in general, and based on the results displayed in §3.3, the following consideration can be made:

- equal percentage parameters variations do not always result in larger variations for longer trains respect to short ones; they actually seem comparable in general;
- the behavior of Traction and Compression forces often differs both quantitatively (minor or major variation) and qualitatively (e.g. the same parameter variation can cause an increase of compression forces while decreasing the traction ones; an example of this behavior can be found observing the LFPD values derived from the variation of the initial pressure in the brake pipe (*initpress*) in Figure 3.3;
- the comparison between Figure 3.3 and Figure 3.4 clearly shows that changing operational parameters (e.g. the breaking maneuver) can have a significant impact on the results of this kind of analyses.;
- any consideration based on the ranking of parameters resulting from this analysis should be avoided, since +/- 30% is an unrealistic variation range for the most of parameters.

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4 Sensitivity Analysis on Technical Parameters

4.1 Objective

Once completed the general exploration of the model, the second step of the sensitivity analysis is focused on the effect of "realistic" uncertainties affecting the technical parameters on simulations outcome.

According to §1.3, the sensitivity analysis is performed in order to identify the technical parameters whose uncertainty mainly affects the LTD simulation outcome(s) and must be taken into account in the LTD simulations performed for the demonstrator trains (i.e. when the focus of the sensitivity analysis will be moved onto the operational parameters). In other words, it allows to identify the technical parameter uncertainties that may be neglected and, in this way, to simplify the workflow of the subsequent analyses.

4.2 Methodology

For this step of the analysis, two reference trains families were considered: 400LL 300GP and 4GP. See §2.2.1 for additional details on these families. The standard and the alternative emergency braking maneuver are considered for the 400LL 300GP family, while only the standard one is considered for 4GP. See §2.2.1 for additional details on these families.

Since the scope of this analysis step was to identify possible technical parameters to neglect, in order to completely investigate the influence of their uncertainties, all order of interactions had to be considered. That is, it is not sufficient to attest that modifying the parameters alone do not result into large LTD differences, also their interactions have to be tested.

The model output considered for the sensitivity analysis is the Lower Force Probability (LFP), introduced in §2.5.2.

With reference to the different techniques developed for sensitivity analysis, briefly introduced in the Appendix A (*Importance and sensitivity analysis*), the "Finite change" approach is adopted. This non-parametric approach allows the apportionment of the model output change into the contributions due to the individual and simultaneous changes of the input variables. Specifically, the "First Order Finite Change Sensitivity Index" and the "Total Order Finite Change Sensitivity Indices" are calculated for each parameter.

Defining $\boldsymbol{x} = (x_1, \dots, x_i, \dots, x_n)$ as the vector of parameters of interest and y as the model output, a general model can be written as $y = G(\boldsymbol{x})$.

Now a positive difference vector $\Delta\boldsymbol{x}^+ = (\Delta x_1^+, \dots, \Delta x_i^+, \dots, \Delta x_n^+)$ and a negative one $\Delta\boldsymbol{x}^- = (\Delta x_1^-, \dots, \Delta x_i^-, \dots, \Delta x_n^-)$ need to be defined where $\Delta x_i^+$and Δx_i^- are respectively the maximum increment and decrement for the i-th parameter that have been established for the sensitivity

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study. These increments were chosen based on the technical uncertainties listed in §1.4.1, where $\Delta x_i = \pm 3\sigma$ was considered.

At this point it is possible to define the first order sensitivity indicator D_i^1 for the parameter i as:

$$D_i^1 = G(x_1, \dots, x_i + \Delta x_i, \dots, x_n) - \mathrm{G}(\boldsymbol{x})$$

While the total order sensitivity indicator D_i^{tot} will be:

$$D_i^{tot} = \mathrm{G}(\boldsymbol{x} + \Delta\boldsymbol{x}) - G(x_1 + \Delta x_1, \dots, x_i, \dots, x_n + \Delta x_n)$$

D_i^1 measures how much varying the i-th parameter alone affects the model outcome, while D_i^{tot} quantifies the total contribute (including those deriving from interactions with other parameters) of the i-th parameter to the global outcome variation obtained when all parameters together are incremented or decremented.

With reference to the LFP, assumed as model output, it results:

$$D_i^1 = LFP_i - 0{,}5 = \mathrm{LFPD}$$
$$D_i^{tot} = \mathrm{LFP_{all}} - \mathrm{LFP_{all\_but\_i}} = (\mathrm{LFP_{all}} - 0{,}5) - (\mathrm{LFP_{all\_but\_i}} - 0{,}5) = \mathrm{LFPD_{all}} - \mathrm{LFPD_{all\_but\_i}}$$

This procedure has been conducted both increasing and decreasing the parameters, for a total of 4N+2 simulations (for each family), where N is the number of parameters to analyze.

4.3 Results

The total number of TrainDy runs for this sensitivity analysis were:

$$(4 \cdot (\sim)20\ parameters + 2) \cdot (3\ families/manouvers \cdot 1000\ trains)\ \approx 246k$$

The huge amount of output data has been post-processed by the python API and to efficiently present the results, two type of plots were generated: CDFs plots and Tornado plots.

CDFs plots contain a high degree of information, but a large number of them is necessary to fully represent the results. Hence, in order to summarize the analysis, the Tornado plots were created.

4.3.1 CDFs plots

Two kinds of CDFs plots were produced for the uncertainty analysis:

- the first one shows the CDF of the reference family and the two resulting from an increase and decrease of a single parameter at time. Figure 4.1 is shown as an example;

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- the second one reports the reference family CDF, the two CDFs obtained by incrementing and decrementing all the technical parameters at the same time and the two CDFs obtained varying all the parameters but the one of interest.

This has been done for all the reference families considered in the analysis and both for traction and compression forces. The obtained plots allow to have qualitative comparison between the different reference family behaviours and to get an insight on the impact of each technical parameter uncertainty.

Figure 4.1 and Figure 4.2 are shown examples. The complete list of plots can be found in the "M2O D2.2 Extended results.pdf" attachment that completes this deliverable as described in the Annex.

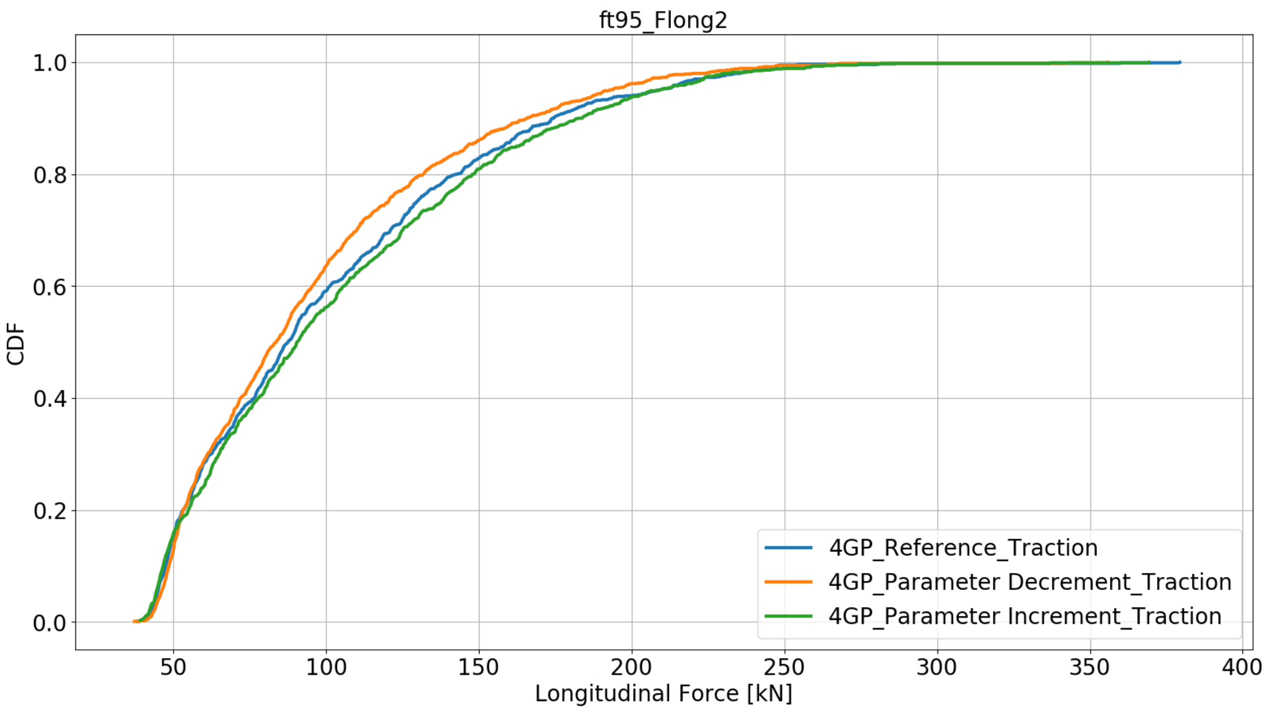

Figure 4.1 – Example of 1st order CDFs plot

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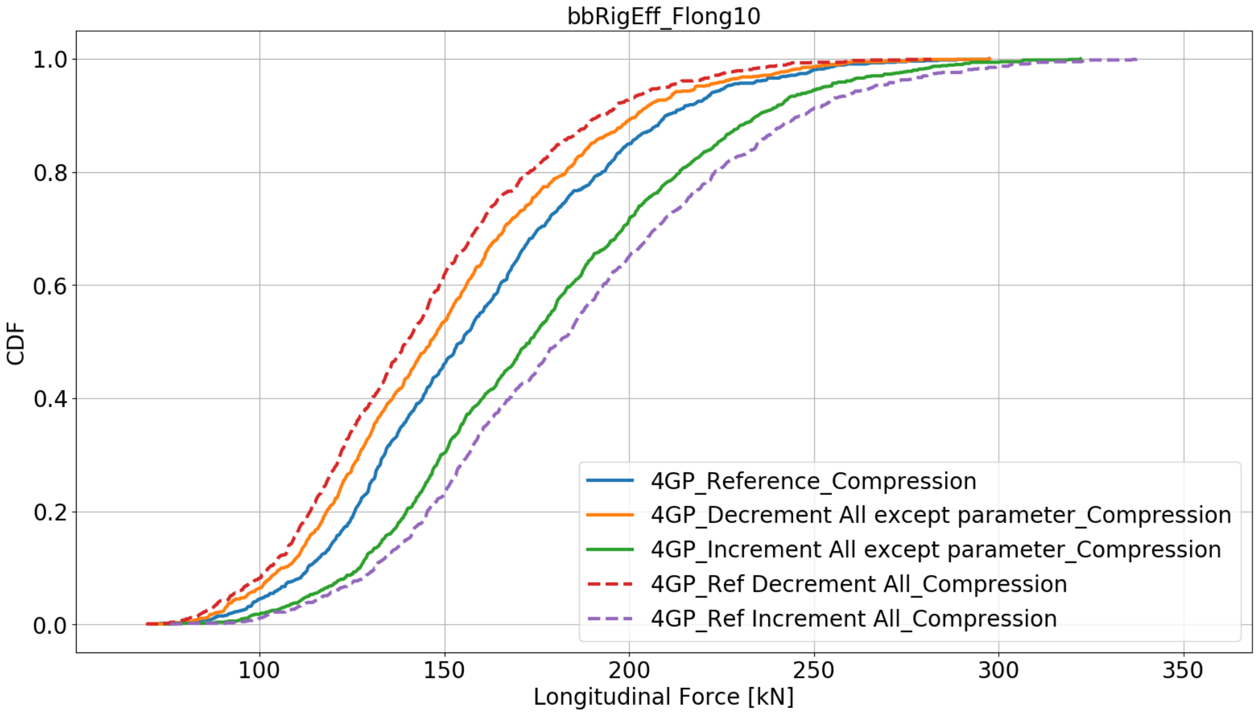

Figure 4.2 – Example of total order CDFs plot

4.3.2 Tornado Plots

Once again, tornado plots allow to summarize the entire analysis result with only a few images; the procedure used to translate the CDFs plot into the tornado chart is the same as the one described in §3.3.2. In this case, a plot is produced for each reference family dividing between traction and compression forces for a total of 6 plots.

Figure 4.3 and Figure 4.4 report the result for the 400LL300GP family, Figure 4.5 and Figure 4.6 the ones for the 400LL300GPN202 and Figure 4.7 and Figure 4.8 the ones for the 4GP. the first order sensitivity indicator D_i^1 is reported in blue, whereas the total order one D_i^{tot} is in red. As shown in §4.2, both these indicators are actually computed using the LFPD indicator.

Since D_i^1 measures the influence of one parameter technical uncertainty alone on the LTD whereas D_i^{tot} the influence of the same parameter when it is varied together with all the other ones, it is clear how the interaction terms (in green) is obtained through a simple difference between the two.

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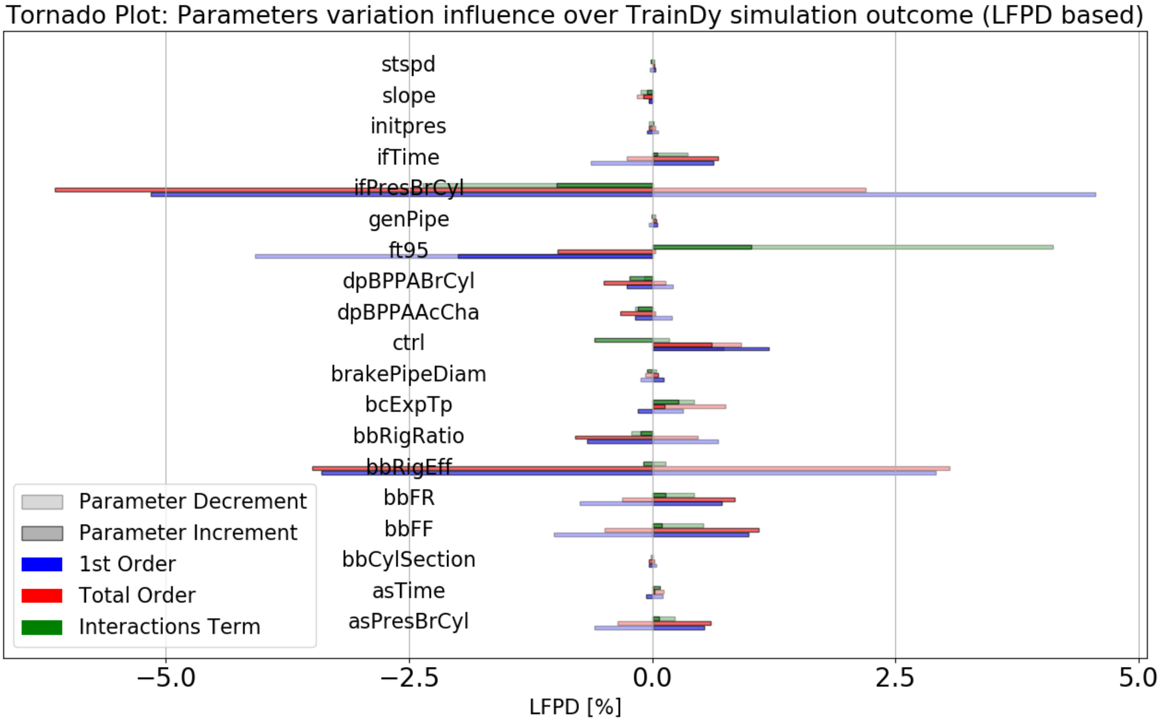

Figure 4.3 - Tornado plot comparing first and total order technical parameter uncertainties influence on traction forces for the 400LL300GP family.

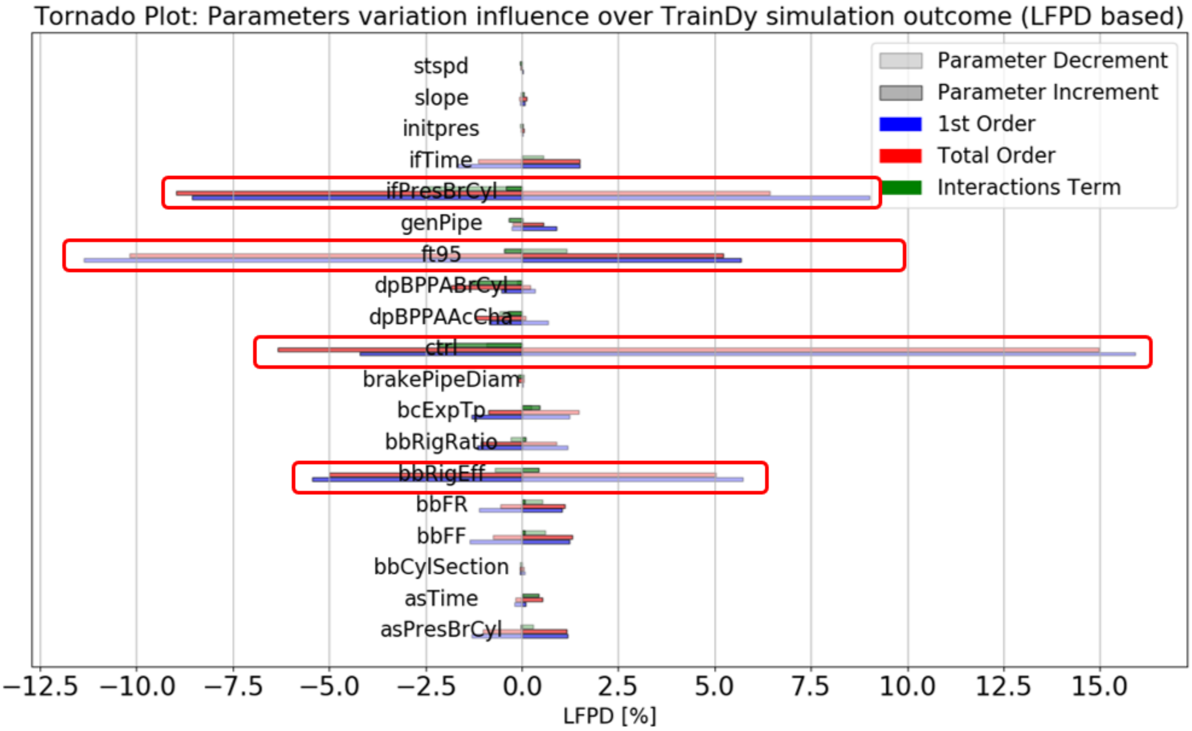

Figure 4.4 - Tornado plot comparing first and total order technical parameter uncertainties influence on compression forces for the 400LL300GP family.

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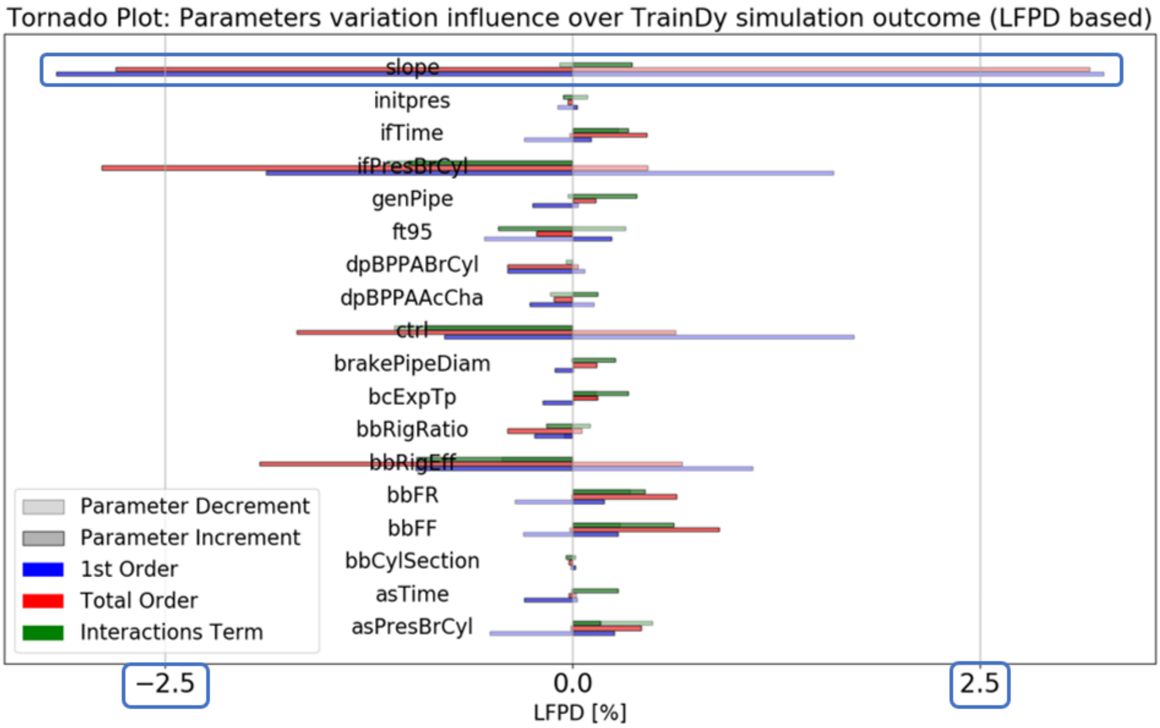

Figure 4.5 - Tornado plot comparing first and total order technical parameter uncertainties influence on traction forces for the 400LL300GPN202 family.

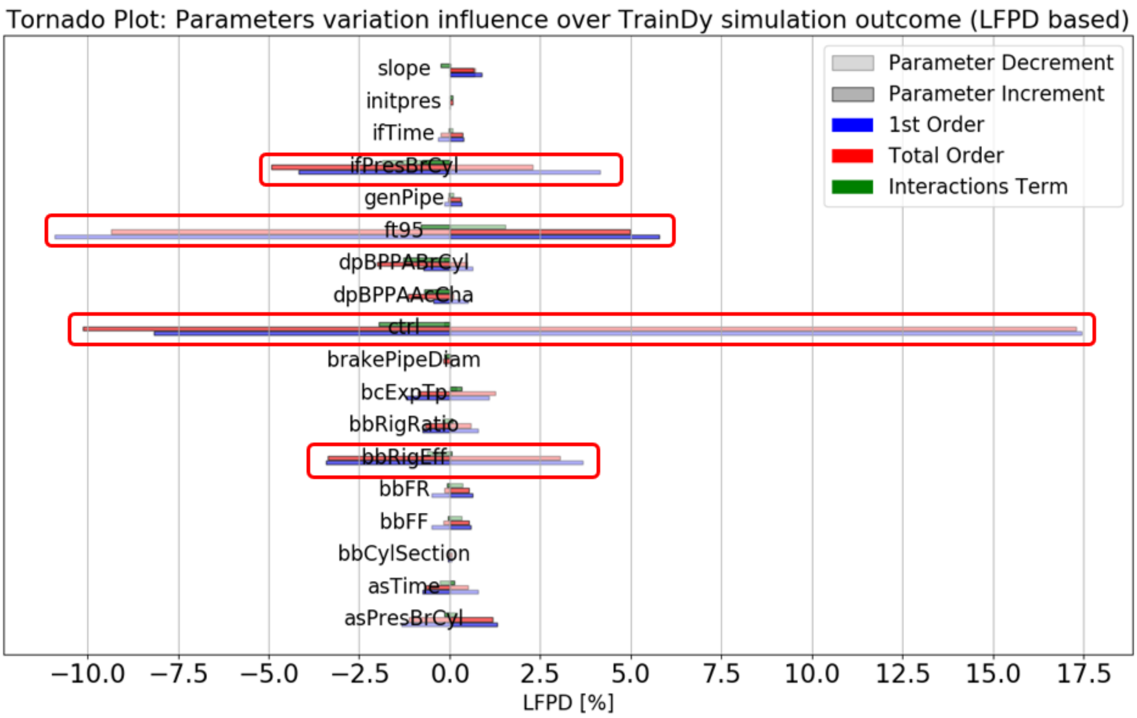

Figure 4.6 - Tornado plot comparing first and total order technical parameter uncertainties influence on compression forces for the 400LL300GPN202 family.

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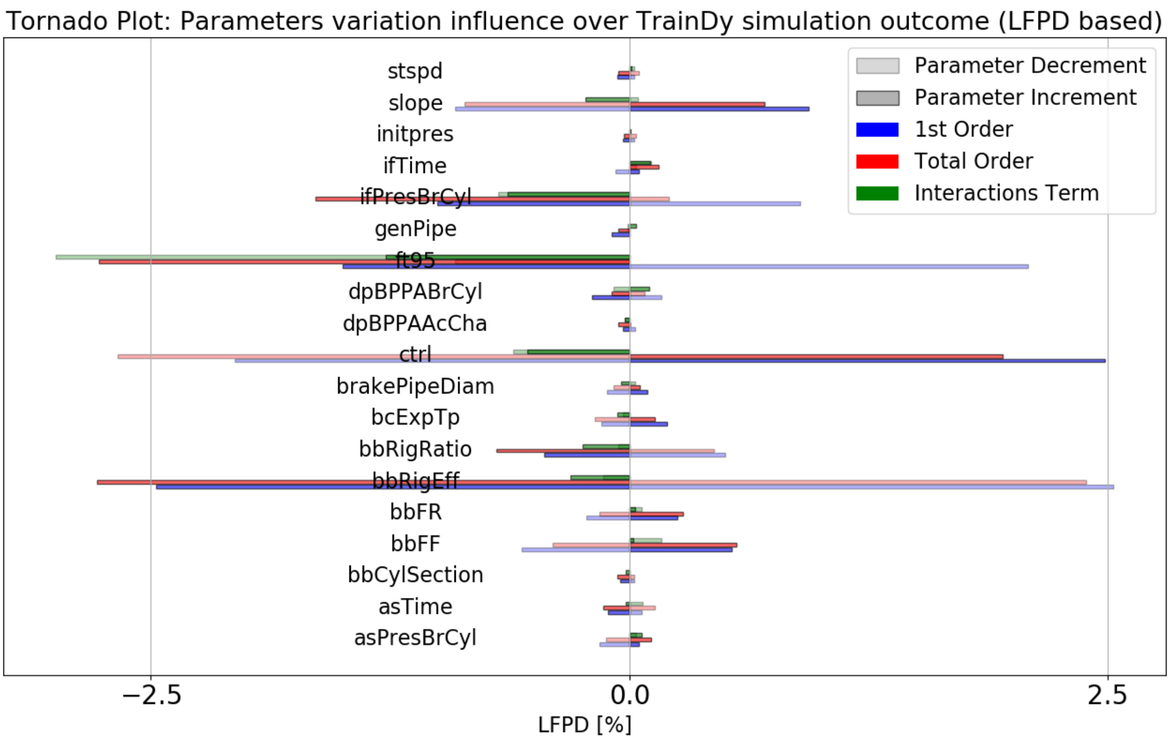

Figure 4.7 - Tornado plot comparing first and total order technical parameter uncertainties influence on traction forces for the 4GP family.

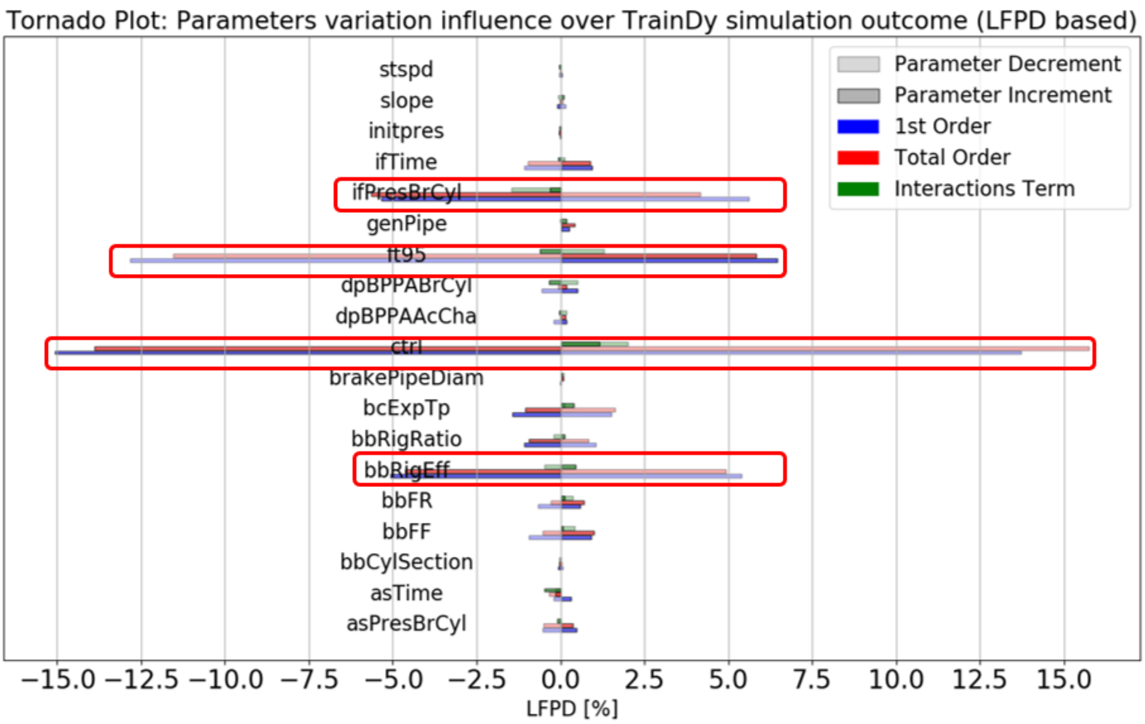

Figure 4.8 - Tornado plot comparing first and total order technical parameter uncertainties influence on compression forces for the 4GP family

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4.3.3 Final Considerations

Based on the results displayed in the above sections, some considerations can be made.

As general considerations concerning the results of the sensitivity analysis:

- The interaction between parameters appears to be pretty small and may be neglected, in fact, the interaction terms are way smaller than the first and total one;
- There are differences among the family responses to the parameters variations but these are limited: changing the maneuver (400LL300GP Vs 400LL300GPN202) does not result in significative changes in terms of parameters importance and neither does the number of coupled trains (400LL300GP Vs 4GP);
- Traction forces are way less affected by parameters variation respect to compression ones.

As specific considerations concerning the addressed parameters:

- The following parameters appear to have a non-negligible technical uncertainty (LFPD > 5%, up to 17,5%):
  - ft95: time needed to fill the braking cylinder at 95% [+/- 25%];
  - ifPresBrCyl: Pressure in brake cylinder for "in-shot function" phase [+/- 10%];
  - bbRigEff: Mean efficiency of the rigging [+/- 8%];
  - ctrl: delay scatter in GSM-R communication [+/- 30%];
- All others parameters have an LFPD < 2,5% except for the "slope" in traction forces of the 400LL300GP case (still significantly below 5% though).

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5 Investigation on trains family population

According to §2.2, the entire sensitivity analysis is performed on families composed by 1000 trains that are randomly sampled from the "real world distribution", that is, the distribution built using the data available regarding the trains currently running on the railway network. It is implicitly assumed that this family is representative of the whole population (for the given outcome).

This section provides the results coming from an additional study performed on this topic, in order to gather insights on the number of trains to be sampled from the whole population, in order to obtain statistically significant results from LTD simulations, i.e. results negligibly affected by the sample size.

The study is focused on the 400LL300GP basic family and, instead of the compression longitudinal forces, the UIC force ratio was used when comparing results (see §2.4 for additional details on it).

The first concern to be addressed was to generate a family that could be assumed to be the reference real world population; it was assumed that a family of 10^5 (i.e. 1e5) trains would have been sufficient. At this point, a TrainDy simulation was run on all the 10^5 trains and the results were sampled in different ways in order to identify what could have been a sample large enough for a reference trains family to be representative of the entire real-world population.

5.1 General family behavior exploration

Preliminarily, the general behavior of the reference family and its samples was studied, both in a qualitative and quantitative way.

The CDFs originated by 100 samples of increasing size are plot in Figure 5.1 and Figure 5.2 together with the 10^5 trains CDF, that is, each line in the plot is the CDF of a smaller family sampled from the reference one. This helps to *qualitatively* understand how increasing the sample size progressively leads all the CDFs to collapse on the 10^5 one.

Looking at the graphs, it appears that a sample of 1000 trains can be considered acceptable in terms of reproducing the general behavior of the family, increasing to 2000 may lead to the best ratio between run-time and quality of the simulation results, whereas increasing to 10000 trains, the CDFs basically collapse into the reference one so it appears that further increasing the sample size does not bring significant advantages.

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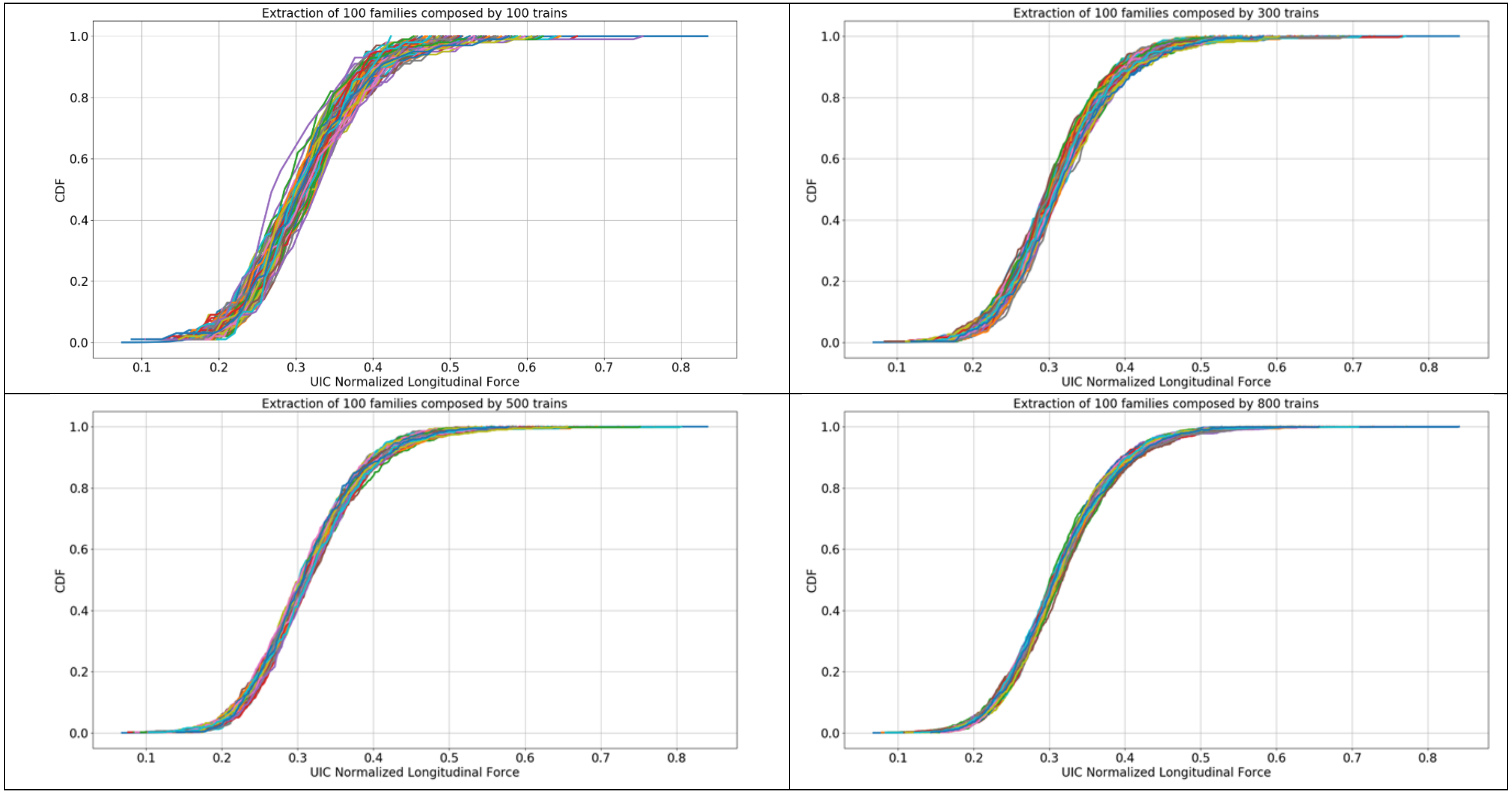

Figure 5.1 - Visualization of 100 CDFs generated from samples of 100, 300, 500 and 800 trains of the reference family Vs the reference family

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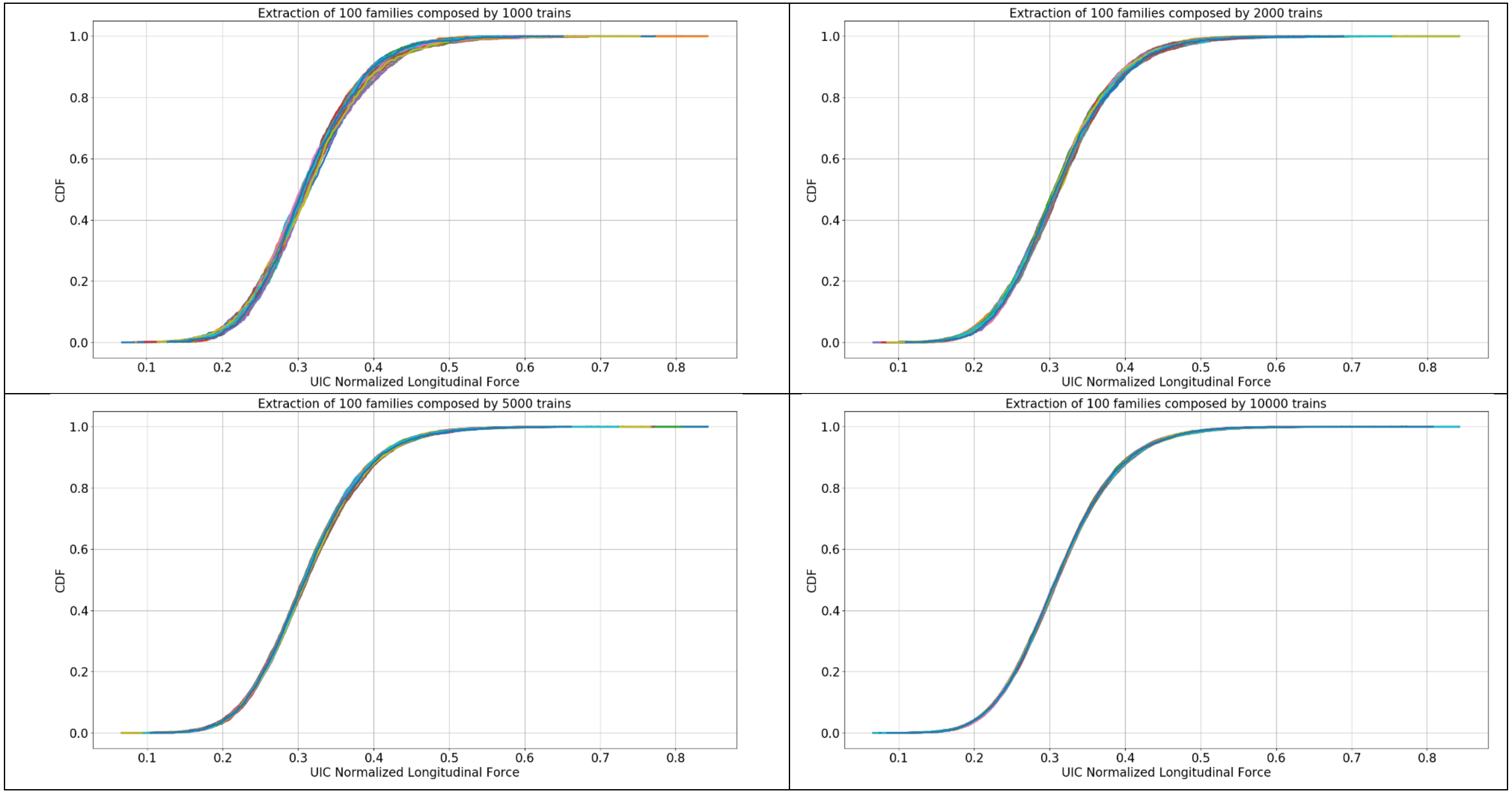

Figure 5.2 - Visualization of 100 CDFs generated from samples of 1000, 2000, 5000 and 10000 trains of the reference family Vs the reference family

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Next, a similar study on global family behavior was conducted, this time though using the LFPD (see §2.5.2) in order to get more quantitative results. Each point in Figure 5.3 is computed by implementing the following procedure:

1) A sample size N is fixed (in the picture N goes from 100 to 5000);
2) 1000 samples of size N are randomly extracted from the 1e5 reference family;
3) The CDF of each sampled family is computed and the LFPD (in absolute value) is calculated respect to reference;
4) Among all the 1000 LFPDs both the maximum value (in blue) and the mean value (in orange) are reported in the graph.

Figure 5.3 – LFPD based comparison between the 10^5 reference family and samples of increasing size.

The quantitative results confirm the qualitative ones: looking at the average value of the LFPD, the general behaviour of the reference family is faithfully represented, on average, even with small samples.

It is interesting also to note that even if the worst extraction among the 1000 done for each sample size is considered, LFPD value rapidly drops under 5% which was the threshold used in §4 to identify significant technical parameters uncertainties to be taken into account during TrainDy simulations. Maximum LFPD value for samples of 1000 trains is around 3%.

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5.2 CDFs tails exploration

For safety applications, in addition to the general behavior, it is important to investigate the tails of the CDFs, especially when they extend beyond the UIC admissible value. For this reason, the same LFPD study described in §5.1 was repeated, this time applying the indicator on a reduced forces range (UIC force ratio > 0.4). The results are shown in Figure 5.4.

Figure 5.4 - LFPD based comparison between the 1e5 reference family and samples of increasing size. The LFPD indicator has been restricted to UIC force ratio > 0.4.

According to Figure 5.4, both the trend of maximum and mean value of the LFPD are worse when a reduced range of forces is considered instead of the entire one. Looking at the mean value, the LFPD rapidly drops when the sample size is increased (at 1000 trains it is already around 2%). Observing the worst-case scenario among the 1000 extracted samples instead (i.e. the blue line), it can be seen that the threshold of 5% is reached only when the sample size is between 2000 and 3000 trains. It is worth notice that these numbers can significantly vary if a different force range is considered. Increasing the ratio end therefore reducing the number of trains that overcome that threshold, the sample size to have a negligible estimation has to increase. Unfortunately, it is not possible to draw a general conclusion for different UIC force ratios or different reference families, but just an insight on the specific problem.

Finally, a last study was performed in order to quantify the uncertainty associated to a CDF result, knowing the size of the sample. Indeed, when the sample size is finalized and TrainDy simulations are run, one of the key information that will have to be extracted is the probability associated to passing the UIC admissible longitudinal force, or vice versa, what is the force associated to a given cumulative probability.

Since it has been showed previously that different samples of the same size will return different CDFs, but only one sample will be used as the representative family for the simulations, this dispersion of probability values (in blue) and force values (in red) have to be understood and managed, as illustrated in Figure 5.5., where an assumed normalized longitudinal force F (i.e. ratio between LCF and admissible LCF) is displayed.

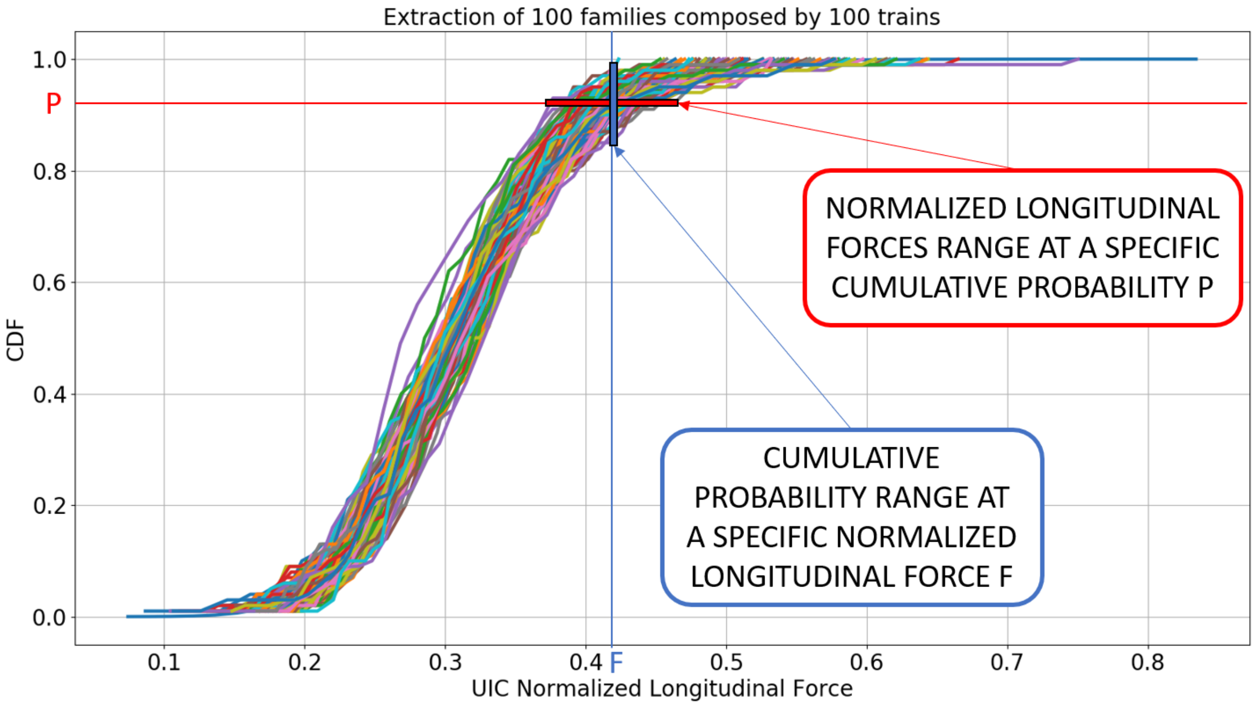

Figure 5.5 – Definition of uncertainties related to CDF results reading

5.2.1 Cumulative probability uncertainty

To clarify the uncertainty on the cumulative probability, each point on the graph reported in Figure 5.6 was computed using the following procedure:

1) a sample size N and a normalized longitudinal force value F are chosen;
2) 1000 samples of size N are extracted from the 10^5 reference family;
3) the CDF of each sampled family is computed and the difference between the cumulative probability associated to F for the sample and for the 10^5 reference family is registered;

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4) the maximum difference probability value among the 1000 samples is reported in the graph.

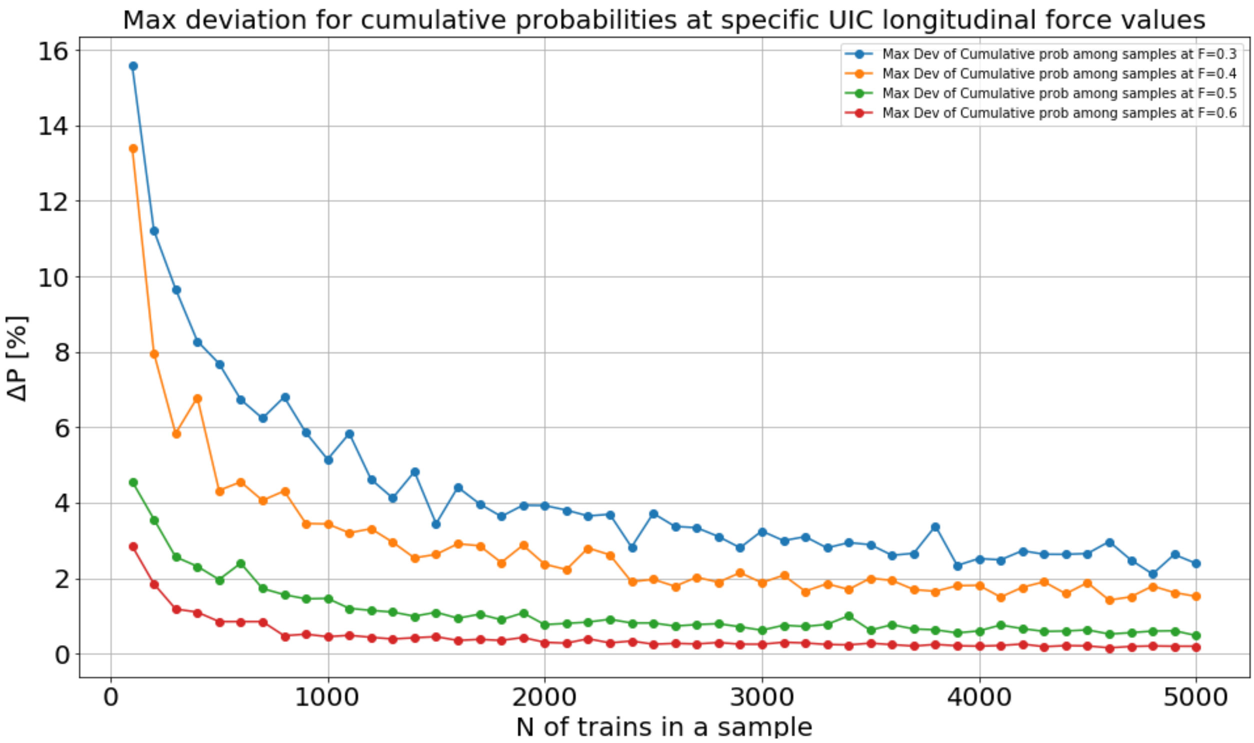

Figure 5.6 – Uncertainty on cumulative probability associated to a specific force value as a function of sample size.

According to Figure 5.6, increasing the sample size reduces the maximum uncertainty associated with the cumulative probability. As an example on how these graphs should be used, choosing 1000 trains as the size of the representative family for simulation, when reading the cumulative probability associated to the UIC force ratio 0.4 (which in the specific case of the 400LL300GP is around 90%) a $\pm$3.5% max uncertainty should be taken into account to be fully conservative. It is important to notice how these uncertainties tend to decrease the higher the UIC force ratio is chosen. This happens because, in general, the CDFs tend to converge more into the tail regions as shown in Figure 5.5.

5.2.2 UIC force ratio uncertainty

To clarify the uncertainty on the normalized longitudinal force F, each point on the graph reported in Figure 5.7 was computed using the following procedure:

1) a sample size N and a cumulative probability P are chosen;
2) 1000 samples of size N are extracted from the 10^5 reference family;
3) the CDF of each sampled family is computed and the difference between the UIC force ratio associated to P for the sample and for the 10^5 reference is registered;

4) the maximum difference of UIC force ratio values among the 1000 samples is reported in the graph.

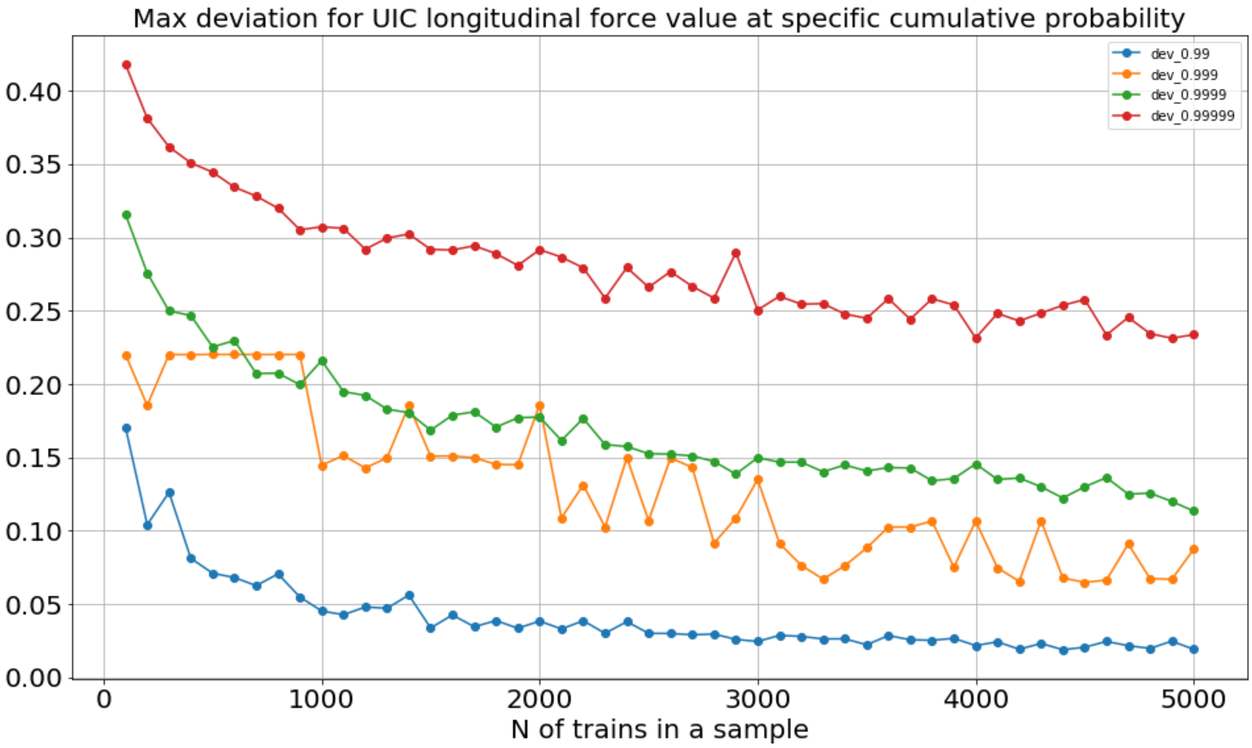

Figure 5.7 - Uncertainty on UIC force ratio associated to a specific cumulative probability as a function of sample size.

According to Figure 5.7, the increasing of the sample size reduces the maximum uncertainty associated with the UIC force ratio reading. As an example on how these graphs should be used, if one was to choose 1000 trains as the size of the representative family for a simulation, when reading the UIC force ratio associated to a cumulative probability of 99.9%, a ±15% max uncertainty should be taken into account to be fully conservative. From these results it appears impractical to reach an uncertainty of less than 5% for cumulative probabilities higher than 99%

5.3 Final considerations

The main conclusions relative to the study conducted on trains family population can be summarized as follow:

1) since the results contained in this section are highly dependent on the reference population, an investigation similar to the one described in §5.1 and §5.2 should be performed every time a new reference family is generated with the scope of conducting extensive LTD analyses. This approach can be followed for the two demonstrators
2) it appears that, in general, the average behavior of a train population can be reproduced with a relatively small sample size. This becomes less and less true when the range of forces on which real population and sample are compared is reduced;
3) plots like the one reported in Figure 5.6 and Figure 5.7 are particularly important since they allow to associate an uncertainty respectively to the cumulative probability and UIC force ratio values, knowing the size of population sample. As previously specified, unfortunately, these uncertainties quantification cannot be generalized to different referent families or maneuvers.

Based on this, the recommended strategy to generate and validate a new reference family should be, first of all, to generate a great number of trains (like 10^5) using the distributions derived by the real-world population in order to be sure to have a good representation of it. Then, it has to be confirmed that the population general behaviour is captured even with a small number of trains (see §5.1).

Finally, especially if the reference population CDF tail reaches a UIC force ratio equal to one (or its proximity), the focus of the investigation should be shifted to a restricted range of higher longitudinal forces. Looking at a similar plot like the one in Figure 5.4, the final sample size chosen should guarantee that the average LFPD value remain at least under 5% (since that is the threshold used for technical parameters uncertainties in §4), while the evaluation of the admissible max LFPD could be different case by case. Indeed, if the CDF tail of the reference population reaches UIC force ratio values that are well below the unity, even a LFPD=10% can be accepted since this is an indicator useful only when looking at a restricted range of forces and even adding this uncertainty to LTD simulations, the risk of crossing UIC force ratio = 1 is practically null. Nevertheless, if this condition was not to be applicable (i.e. the population CDF tail reaches the proximity of one), the sample size should be increased in order to reduce the max LFPD value. If that was the case, it can be especially useful to produce a plot like in Figure 5.6 relative to a UIC force ratio equal to one. That would give the user a good estimation of the maximum uncertainty on the TrainDy calculated probability that the consist may derail.

Taking into account all these considerations, it was shown that 1000 trains were a sufficient sample

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size to generate a 400LL300GP reference family.

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6 Conclusion

The activities performed during the M2O project concerning the sensitivity analysis on the TrainDy software and the results obtained by these activities are reported in the present deliverable. The final objective of the performed activities is to support the simulations that will validate the Longitudinal Train Dynamic (LTD) of the demonstrator trains (in deliverable 3.3).

A preliminary analysis have been performed on the behavior of the TrainDy model outcomes, assuming a fictitious uncertainty (i.e. finite change) of each Technical parameter.

Different (parametric and non-parametric) approaches for sensitivity analysis were addressed. A specific sensitivity measure (Lower Force Probability Differential) is introduced, accounting for the peculiarities of the LTD simulations (whose outcomes are CDFs referred to trains family from the whole population). The "Finite change" approach is adopted, generally computing the First Order and the Total Order Finite Change Sensitivity Indices for each input parameter.

This approach has been applied to a subset of parameters affecting the LTD, and specifically to the Technical parameters (see §1.4.1). Specifically, a complete sensitivity analysis has been performed for (two) families (400LL 300GP, 4GP), based on justified assumptions on the uncertainty assigned to parameters. As main results of the sensitivity analysis performed on Technical parameters, the TrainDy model outcomes are significantly affected only by a limited subset of parameters. **These "critical" Technical parameters, and the related uncertainties, should be considered in the LTD simulations performed for the demonstrator trains since they should provide evidence that, even taking the technical uncertainties into account, the consists may run safely.**

The approach here proposed for sensitivity analysis, based on the "Finite change" of the input parameters and on the computation of the Lower Force Probability, could be adopted in the LTD simulations for the demonstrator trains will refer to the Operational parameters (see §1.4.2).

An additional study has been performed on the number of trains to be sampled from the whole population, in order to obtain statistically significant results from LTD simulations. **A procedure that allows to individuate the sample size needed to create a new reference trains family has been proposed (see §5.3)** and evidence that 1000 trains were enough for the 400LL300GP reference family were given.

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Appendix A Importance and sensitivity analysis

The Importance and Sensitivity analysis aims at quantifying the contribution of the input variables to the model output (Importance analysis) and to the related uncertainty (Sensitivity analysis). They allow the ranking of the input variables and give information about the "direction" of the model output change due to the "one at time"[5] and simultaneous changes of the input variables, the key-drivers of the change and the structure of the model.

Different techniques developed for sensitivity analysis can be classified into two main branches, depending on the problem setting [A_ 9]: Global analysis, focused on the uncertainty on the model output with reference to the entire range of values of the input variables; Local analysis, focused on the uncertainty on the model output with reference to the values of the input variables.

Local measures deal with a point value of the model output and input variables. They cannot be used for finite changes of the input variables or, in this case, they do not include the contributions of non-linear terms (i.e. interactions among input variables, whose effects are manifested for their simultaneous changes and are not taken into account by the super-imposition of the effect due to the one at time change of variables). Moreover, they are not "additive": the measure for a group of input variables cannot be computed as the sum of the measures estimated for each single variable but requires new evaluations of the model.

The approaches recently proposed for Importance and sensitivity analysis refer to two different representations of the model output: Taylor series representation [A_ 3]; High Dimensional Model Representation (HDMR) [A_ 4], [A_ 7], [A_ 8], [A_ 9].

IMPORTANCE AND SENSITIVITY MEASURE FROM HDMR REPRESENTATION

According to HDMR, the model output can be written as sum of terms which depend on an increasing number of input variables [A_ 4]. The constant term is the average value of the model output. The firs-order terms, which depend on single variables, are the "Main effects"; each term is obtained by the difference between the model output when all variables change in their range of values but one (which is fixed) and the average value. The subsequent terms are named "High order interaction"; each term depends on two or more variables (according to the order) and is obtained by the difference between the model output when all variables change in their range of values but the ones at issue (which are fixed) and all the lower-order terms.

Starting from the HDMR, two different approaches are proposed in the applicable scientific literature: a parametric approach for the estimation of the "Variance-based sensitivity indices" [A_ 8], [A_ 9]; a non-parametric approach for the estimation of the "Finite change sensitivity indices" [A_ 7].

[5] One variable changes while the remaining ones are fixed to their values.

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Variance based approach

The "Variance-based" approach for Sensitivity analysis allows the apportionment of the variance on the model output into the contributions due to the variance on input variables. This parametric approach assumes that uncertainty is specified by a normal probability distribution.

Squaring HDMR and using the orthogonality property among its terms, the total variance on the model output can be written as the sum of terms (partial variance) depending on an increasing number of variables (Variance decomposition). The ratios between the partial variance and the total variance are the "Sensitivity indices". The sum of these non-negative terms is equal to one.

The computation of the sensitivity indices requires the solution of multi-dimensional integrals by sampling-based methods (e.g. Monte Carlo) or the application of the Fourier transform [A_ 8].

Finite change approach

The "Finite change" approach for Importance and Sensitivity analysis allows the apportionment of the model output change into the contributions due to the individual and simultaneous changes of the input variables. This non-parametric approach does not require the specification of a probability distribution to describe the uncertainty on the input variable. [A_ 5] [A_ 6]

Starting from the High dimensional model representation, the finite change of the model output is decomposed into $2^n - 1$ terms (where n is the number of input variables) which depend on an increasing number of input variables. The following indices are defined:

- the "First Order Finite Change Sensitivity Index", which is the contribution to the finite change of the model output due to the finite change of a single input variable; its sign gives information on the direction of change.
- the "Order k Finite Change Sensitivity Index" , which is the contribution to the finite change of the model output due to the interactions among (the first) k variables; its sign indicates whether the interactions result in "cooperation" (contribution to finite change >0) or "interference" (contribution to finite change < 0);
- the "Total Order Finite Change Sensitivity Indices" , which is the contribution to the finite change of the model output due to the finite change of a single variable alone and together with the changes of all remaining variables in any number and combination.

The Total Order Sensitivity Indices allow the identification of the key-drivers of the model output change. The first and higher order Sensitivity Indices give information about the structure of the model. The computation of all order Indices can be performed directly their definitions, requiring 2^n evaluations of the model's outcome. If the complete decomposition is not achievable due to computational cost, the differences between the Total Order (requiring n+2 outcome evaluations) and the First Order (requiring n+2 outcome evaluations) Finite Change Sensitivity Index one can be taken as indicators; if this difference is closed to 0, the effects of interactions are irrelevant; otherwise, the input variable is relevant also for its cooperation with the others ones.

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Appendix B Mathematical description of LFP computing

This appendix contains analytical proofs linked to the Lower Forces Probability (LFP).

§B.1 and §B.2 delivers two different methods to compute such probability, §B.3 extends the definition to when a specific range of forces is considered instead of the whole spectrum and finally §B.4 analytically derives the LFP values when a single distribution is considered.

B.1. Mathematical proof for LFP derivation (Method 1)

Definitions

- $P_r(f)$: Probability Distribution Function (PDF) for a train extracted from the reference family to present a specific longitudinal force.
- $P_t(f)$: Probability Distribution Function (PDF) for a train extracted from the target family to present a specific longitudinal force.
- F_r : Aleatory variable that represent the longitudinal force assumed by a train randomly extracted using $P_r(f)$.
- F_t : Aleatory variable that represent the longitudinal force assumed by a train randomly extracted using $P_t(f)$.
- $F_x = F_r - F_t$

Analytical description

If $P_r(f)$ and $P_t(f)$ were continuous one could define the probability that F_x assumes a specific value using the convolution integral:

$$P(F_x = x) = \int_{-\infty}^{+\infty} P_r(f)P_t(f-x)df = \int_{f_{min}}^{f_{max}} P(F_r = f)P(F_t = f-x)df$$

Where f_{min} , and f_{max} determine the force range limit in which both $P_r(f)$ and $P_t(f-x)$ are different from zero.

At this point $P(F_x = x)$ needs to be calculated for a sufficient number of x in order to build per points the $P_x(f)$ distribution.

Finally, the $CDF_x(f)$ can be obtained integrating $P_x(f)$ and the probability that a randomly extracted target train would present a lower longitudinal force than a reference one will be:

$$\boxed{P(F_x > 0) = 1 - P(F_x \leq 0) = 1 - CDF_x(0)}$$

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B.2. Mathematical proof for LFP derivation (Method 2)

Definitions

- $P_R(f)$: Probability Distribution Function (PDF) for a train extracted from the reference family to present a specific longitudinal force.
- $P_T(f)$: Probability Distribution Function (PDF) for a train extracted from the target family to present a specific longitudinal force.
- $CDF_R(x) = \int_{-\infty}^{x} P_R(f)df$: Cumulative Distribution Function (CDF) for reference trains forces
- $CDF_T(x) = \int_{-\infty}^{x} P_T(f)df$: Cumulative Distribution Function (CDF) for target trains forces

Analytical description

Let us consider to extract two trains, one from the reference family distribution and the other from the target one. Selecting a specific longitudinal force f, the probability that the reference train will present a force in the neighborhood of f will be equal to $P_R(f)df$, while the probability that the target train will present lower forces than f is $\int_{-\infty}^{f} P_T(f')df'$. Hence, the probability that considering a specific force, the target train will present lower forces than the reference one is:

$$P_R(f)df \int_{-\infty}^{f} P_T(f')df' = P_R(f)CDF_T(f)df$$

Integrating this expression on all the possible forces that the reference train can present exactly delivers the probability that extracting two random trains from the different families, the target one will present smaller forces than the reference one:

$$\int_{f_{min}^R}^{f_{max}^R} P_R(f)CDF_T(f)df$$

It is useful to express this integral using only the CDFs of the families instead of their PDFs since CDFs are easier to derive experimentally. To accomplish that, a change in the integration variable is needed: $y(f) = CDF_R(f)$, $dy = d\big(CDF_R(f)\big) = P_R(f)df$. Substituting the new variable in the integral gives:

$$\boxed{\int_{f_{min}^R}^{f_{max}^R} P_R(f)CDF_T(f)df = \int_{y(f_{\min}^R)}^{y(f_{\max}^R)} CDF_T(y)dy = \int_{CDF_R(f_{\min}^R)}^{CDF_R(f_{\max}^R)} CDF_T(y)dy}$$

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B.3. *Restriction of the LFP on a specific range of forces*

If §B.1 or §B.2 definitions are used, the LFP indicator averages the behavior of the two CDFs on the entire force's spectrum. If it is necessary to restrict the information to a specific forces range, additional manipulations are needed.

Let us still consider to extract a reference and a target train, but this time imagining to refuse the extraction if both trains are not in a specific range of forces $\Delta f_{critical} = [f_{min}^C, f_{max}^C]$.

The probability that the reference train will present a specific longitudinal force in the neighborhood of f (where $f \in \Delta f_{critical}$) it is still equal to $P_R(f)df$, while the probability that the target train will present lower forces than f <u>and</u> that it has been extracted in $\Delta f_{critical}$ will be $\int_{f_{min}^C}^{f} P_T(f')df'$.

Hence, the probability that considering a specific force inside $\Delta f_{critical}$, the target train will present lower forces than the reference one is:

$$P_R(f)df \int_{f_{min}^C}^{f} P_T(f')df' = P_R(f)df\left[CDF_T(f) - CDF_T\left(f_{min}^C\right)\right]$$

Integrating on all $f \in \Delta f_{critical}$ we obtain:

$$\int_{f_{min}^C}^{f_{max}^C} P_R(f)df\left[CDF_T(f) - CDF_T\left(f_{min}^C\right)\right]$$
$$= \int_{f_{min}^C}^{f_{max}^C} P_R(f)CDF_T(f)df - CDF_T\left(f_{min}^C\right)\int_{f_{min}^C}^{f_{max}^C} P_R(f)df$$

Repeating the same variable change $y(f) = CDF_R(f)$ for the first member of the expression and rearranging the second term using the CDF definition, the following expression can be obtained (assuming $f_{max}^C = f_{max}^R$):

$$\int_{CDF_R(f_{\min}^C)}^{CDF_R(f_{\max}^C)} CDF_T(y)dy - CDF_T\left(f_{min}^C\right)\left[CDF_R(f_{\max}^C) - CDF_R(f_{\min}^C)\right]$$ [6]

If $f_{max}^C = f_{max}^R$ then it will be $CDF_R(f_{\max}^C) = 1$ and the expression can be further simplified as:

[6] At a closer look, this is equal to the probability that the target train present lower forces than a reference one if the target is extracted from the whole distribution while the reference only in $\Delta f_{critical}$ <u>minus</u> the probability that a target train can be extracted outside the critical range while the reference is inside.

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$$\int_{CDF_R(f_{\min}^C)}^{CDF_R(f_{\max}^C)} CDF_T(y)dy - CDF_T(f_{min}^C) + CDF_T(f_{min}^C)CDF_R(f_{\min}^C)$$

What we just calculated is the probability that the two trains are extracted in the critical forces range while the target train present lower forces than the reference.

What is really needed though is the conditional probability that once the trains are extracted in that range (event A) the target train will present lower forces than the reference one (event B).

In other words, we just calculated P(A∩B), while what is really needed is:

$$P(B|A) = \frac{\mathrm{P(A \cap B)}}{P(B)}$$

Since $P(B) = \left(1 - CDF_T\left(f_{min}^C\right)\right) \cdot \left(1 - CDF_R\left(f_{min}^C\right)\right)$, then the LFP modified will be:

$$\boxed{\frac{\int_{CDF_R(f_{\min}^C)}^{CDF_R(f_{\max}^C)} CDF_T(y)dy - CDF_T(f_{min}^C) + CDF_T(f_{min}^C)CDF_R(f_{\min}^C)}{\left(1 - CDF_T\left(f_{min}^C\right)\right) \cdot \left(1 - CDF_R\left(f_{min}^C\right)\right)}}$$

B.4. LFP values if a single distribution is used

LFP computed on whole Forces spectrum

Let us consider that $P_R(f) = P_T(f) = P(f)$

Then, the LFP for two identical distribution will be:

$$\int_{f_{min}}^{f_{max}} P(f)CDF(f)df = [CDF(f)CDF(f)|_{f_{min}}^{f_{max}} - \int_{f_{min}}^{f_{max}} CDF(f)P(f)df$$

$$2\int_{f_{min}}^{f_{max}} P(f)CDF(f)df = 1 - 0$$

$$\boxed{\int_{f_{min}}^{f_{max}} P(f)CDF(f)df = 0{,}5}$$

LFP computed on specific forces range

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Let us consider that $P_R(f) = P_T(f) = P(f)$, and a range of critical forces $\Delta f_{critical} = [f_{min}^C, f_{max}^C]$.

The LFP numerator for two trains extracted from the same distribution will be:

$$\int_{CDF(f_{\min}^C)}^{CDF(f_{\max}^C)} CDF(y)dy - CDF(f_{min}^C) + CDF(f_{min}^C)CDF(f_{min}^C)$$

The first term, following the steps of the previous demonstration, reduces to:

$$\int_{CDF(f_{\min}^C)}^{CDF(f_{\max}^C)} CDF(y)dy = \frac{[CDF(f)CDF(f)|_{f_{min}^C}^{f_{max}^C}}{2} = \frac{1 - CDF(f_{min}^C)^2}{2}$$

Substituting in the previous expression gives:

$$\frac{1}{2} - CDF(f_{min}^C) + \frac{1}{2}CDF(f_{min}^C)^2$$

So the LFP will be:

$$\boxed{\frac{\frac{1}{2} - CDF(f_{min}^C) + \frac{1}{2}CDF(f_{min}^C)^2}{\left(1 - CDF(f_{min}^C)\right) \cdot \left(1 - CDF(f_{min}^C)\right)} = \frac{0.5 \cdot (1 - CDF(f_{min}^C))^2}{(1 - CDF(f_{min}^C))^2} = 0.5}$$

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Annex

This report is completed by the complete list of CFD plots produced by the sensitivity analysis, specifically during the:

- Preliminary exploration of TrainDy model (see §3);
- Sensitivity Analysis on Technical Parameters (see §4).

Such plots can be found in the document attached to this deliverable named "M2O D2.2 Extended results.pdf"

The following is the attachment structure:

- Preliminary exploration of TrainDy model behaviour – CDF plots
  - SRGH Vs 400LL 300GP
    - Compression forces
    - Traction forces
  - N202 SRGH Vs n202 400LL 300GP
    - Compression longitudinal force
    - Traction longitudinal forces
- Sensitivity analysis on technical parameters uncertainty effect – CDF plots
  - 400LL 300GP
    - First Order analysis
      - Traction forces
      - Compression forces
    - Total Order analysis
      - Traction forces
      - Compression forces
  - N202 400LL 300GP
    - First Order analysis
      - Traction forces
      - Compression forces
    - Total Order analysis
      - Traction forces
      - Compression forces
  - 4GP
    - First Order analysis
      - Traction forces
      - Compression forces
    - Total Order analysis
      - Traction forces
      - Compression forces